1. Evaluate $\nabla \cdot F$ and $\nabla \times F$ for each of the following vector fields.
   (a) $F = x \hat{i} + y \hat{j} + z \hat{k}$
   (b) $F = x y^2 \hat{i} - y z^2 \hat{j} + z x^2 \hat{k}$
   (c) $F = \frac{x \hat{i} + y \hat{j}}{\sqrt{x^2 + y^2}}$ (the polar basis vector $\hat{r}$ in 2d)
   (d) $F = \frac{y \hat{i} + x \hat{j}}{\sqrt{x^2 + y^2}}$ (the polar basis vector $\hat{\theta}$ in 2d)

2. Does $\nabla \times F$ have to be perpendicular to $F$?

3. Verify the vector identities
   (a) $\nabla \cdot (f F) = f \nabla \cdot F + F \cdot \nabla f$
   (b) $\nabla \cdot (F \times G) = G \cdot (\nabla \times F) - F \cdot (\nabla \times G)$
   (c) $\nabla^2 (f g) = f \nabla^2 g + 2 \nabla f \cdot \nabla g + g \nabla^2 f$

4. A rigid body rotates at an angular velocity of $\Omega$ rad/sec about an axis that passes through the origin and has direction $\hat{a}$. When you are standing at the head of $\hat{a}$ looking towards the origin, the rotation is counterclockwise. Set $\Omega = \Omega \hat{a}$.
   (a) Show that the velocity of the point $r = (x, y, z)$ on the body is $\Omega \times r$.
   (b) Evaluate $\nabla \times (\Omega \times r)$ and $\nabla \cdot (\Omega \times r)$, treating $\Omega$ as a constant.

5. (Optional — not to be handed in) Find the speed of the students in a classroom located at latitude 49° N due to the rotation of the Earth. Ignore the motion of the Earth about the Sun, the Sun in the Galaxy and so on. The radius of the Earth is 6378 km.

6. Find, if possible, a vector field $A$ that has $\hat{k}$ component $A_3 = 0$ and that is a vector potential for
   (a) $F = (1 + yz) \hat{i} + (2y + zx) \hat{j} + (3z^2 + xy) \hat{k}$
   (b) $G = yz \hat{i} + xz \hat{j} + xy \hat{k}$

7. (Optional — not to be handed in) Suppose that the vector field $F$ obeys $\nabla \cdot F = 0$ in all of $\mathbb{R}^3$. Let $r(t) = tx \hat{i} + ty \hat{j} + tz \hat{k}$, $0 \leq t \leq 1$ be a parametrization of the line segment from the origin to $(x, y, z)$. Define
   $$G(x, y, z) = \int_0^1 t F(r(t)) \times \frac{dr}{dt}(t) \, dt$$
   Show that $\nabla \times G = F$ throughout $\mathbb{R}^3$.

Reminder: Midterm II is scheduled for Wednesday, March 14.