# MATH 317 PROBLEM SET X

Never due

1) Consider the vector field

$$\vec{V}(x,y) = (e^x \cos y + x^2, x^2 y + 3)$$

Evaluate the line integral of $\vec{V}$ around the oriented curve $C$ obtained by moving from $(0,0)$ to $(1,0)$ to $(1,\pi)$ and finally to $(0,\pi)$ along straight line segments.

2) The sides of a grain silo are described by the portion of the cylinder $x^2+y^2=1$ where $0 \le z \le 1$. The top of the silo is given by the portion of the sphere $x^2+y^2+z^2=2$ lying within the cylinder and above the $xy$–plane. Find the flux of the vector field

$$\vec{V}(x,y,z) = (x^2yz, yz + e^x z, x^2 + y)$$

out of the silo.

3) Let $C$ be the curve of intersection of the parabolic cylinder $x = y^2$ and the hyperbolic paraboloid $3z = 2xy$.
   a) Write a vector parametric equation for $C$ using $x$ as the parameter.
   b) Find the length of the part of $C$ between the origin and the point $(9,3,18)$.
   c) A particle moves along $C$ in the direction for which $x$ is increasing. If the particle moves with constant speed 9, find its velocity vector when it is at the point $(1,1,\frac{2}{3})$.

4) Let $\vec{F} = 6x^2yz^2\hat{\boldsymbol{\imath}} + (2x^3z^2 + 2y - xz)\hat{\boldsymbol{\jmath}} + 4x^3yz\hat{\mathbf{k}}$ and let $\vec{G} = yz\hat{\boldsymbol{\imath}} + xy\hat{\mathbf{k}}$.
   a) For what value of the constant $\lambda$ is the vector field $\vec{H} = \vec{F} + \lambda\vec{G}$ conservative on 3-space?
   b) Find a scalar potential $\phi(x,y,z)$ for the conservative field $\vec{H}$ referred to in part (a).
   c) Find $\int_C \vec{F}\cdot d\vec{r}$ if $C$ is the curve of intersection of the two surfaces $z = x$ and $y = e^{xz}$ from the point $(0,1,0)$ to the point $(1,e,1)$.

5) Let $S$ be the part of the surface $z = xy$ lying inside the cylinder $x^2+y^2=3$. Find the moment of inertia of $S$ about the $z$-axis, that is,

$$I = \iint_S (x^2+y^2)\, dS$$

6) Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F} = x^2y^2\hat{\boldsymbol{\imath}} + 2xy\hat{\boldsymbol{\jmath}}$ and $C$ is the boundary of the square in the $xy$–plane having one vertex at the origin and diagonally opposite vertex at the point $(3,3)$, oriented counterclockwise.

7) Let $\vec{F} = -ye^z\hat{\boldsymbol{\imath}} + x^3\cos z\hat{\boldsymbol{\jmath}} + z\sin(xy)\hat{\mathbf{k}}$, and let $S$ be the part of the surface $z = (1-x^2)(1-y^2)$ that lies above the square $-1 \le x \le 1, -1 \le y \le 1$ in the $xy$-plane. Find the flux of $\vec{\nabla}\times\vec{F}$ upward through $S$.

8) Let $B$ be the ball of volume $V$ centered at the point $(x_0,y_0,z_0)$, and let $S$ be the sphere that is the boundary of $B$. Find the flux of $\vec{F} = x^2\hat{\boldsymbol{\imath}} + xy\hat{\boldsymbol{\jmath}} + (3z - yz)\hat{\mathbf{k}}$ outward (from $B$) through $S$.

9) Evaluate the integral $\oint_C \vec{F}\cdot d\vec{r}$, in which $\vec{F} = (e^{x^2} - yz, \sin y - yz, xz + 2y)$ and $C$ is the triangular path from $(1,0,0)$ to $(0,1,0)$ to $(0,0,1)$ to $(1,0,0)$.

**Reminder:** The final exam is on Tuesday, April 15 at 3:30pm in HEBB TH.