

MATH 317 PROBLEM SET I

Due Wednesday, January 15

- 1) Find the velocity, speed and acceleration at time t of the particle whose position is $\vec{r}(t)$. Describe the path of the particle.

a) $\vec{r}(t) = a \cos t \hat{i} + a \sin t \hat{j} + ct \hat{k}$

b) $\vec{r}(t) = a \cos t \sin t \hat{i} + a \sin^2 t \hat{j} + a \cos t \hat{k}$

- 2) A projectile falling under the influence of gravity and slowed by air resistance proportional to its speed has position satisfying

$$\frac{d^2\vec{r}}{dt^2} = -g\hat{k} - \alpha\frac{d\vec{r}}{dt}$$

where α is a positive constant. If $\vec{r} = \vec{r}_0$ and $\frac{d\vec{r}}{dt} = \vec{v}_0$ at time $t = 0$, find $\vec{r}(t)$. (Hint: Define $\vec{u}(t) = e^{\alpha t} \frac{d\vec{r}}{dt}(t)$ and substitute $\frac{d\vec{r}}{dt}(t) = e^{-\alpha t} \vec{u}(t)$ into the given differential equation to find a differential equation for \vec{u} .)

- 3) A gun fires a shell with a muzzle velocity of 150 m/s. While the shell is in the air, it experiences a downward (vertical) gravitational acceleration of 9.8 m/s² and an eastward (horizontal) Coriolis acceleration of 5cm/s². Air resistance may be ignored. The target is 1500 m due north of the gun and both the gun and target are at sea level. For which initial velocities will the shell hit the target?
- 4) Find the specified parametrization of the first quadrant part of the circle $x^2 + y^2 = a^2$.
- a) In terms of the y coordinate.
- b) In terms of the angle between the tangent line and the positive x -axis.
- c) In terms of the arc length from $(0, a)$.
- 5) Find the length of the parametric curve

$$x = a \cos t \sin t \quad y = a \sin^2 t \quad z = bt$$

between $t = 0$ and $t = T > 0$.

- 6) Reparametrize the curve

$$\vec{r}(t) = a \cos^3 t \hat{i} + a \sin^3 t \hat{j} + b \cos 2t \hat{k}, \quad 0 \leq t \leq \frac{\pi}{2}$$

with the same orientation, in terms of arc length measured from the point where $t = 0$.

- 7) The plane $z = 2x + 3y$ intersects the cylinder $x^2 + y^2 = 9$ in an ellipse. Find a parametrization of the ellipse. Find the circumference of this ellipse. You need not evaluate the integral.
- 8) A wire of total length 1000 cm is formed into a flexible coil that is a circular helix. If there are 10 turns to each centimeter of height and the radius of the helix is 3 cm, how tall is the coil?