Math 317 Skeleton Cheat Sheet

Wednesday 2003-04-09

Formulas for ds and $d\mathbf{r}$ $\quad (ds = |d\mathbf{r}|)$

For a parametrized curve $\mathbf{r}(u)$

$$d\mathbf{r} = \frac{d\mathbf{r}}{du}\, du$$
$$ds = \left|\frac{d\mathbf{r}}{du}\right| du$$

For a graph $y = f(x)$

$$d\mathbf{r} = (1, f_x)\, dx$$
$$ds = \sqrt{1 + f_x^2}\, dx$$

Formulas for dS and $\hat{n}dS$ $\quad (dS = |\hat{n}dS|)$

For a parametrized surface $\mathbf{r}(u, v)$

$$\hat{n}dS = \pm\, \mathbf{r}_u \times \mathbf{r}_v\, dudv$$
$$dS = \left|\mathbf{r}_u \times \mathbf{r}_v\right| dudv$$

For a graph $z = f(x, y)$

$$\hat{n}dS = \pm\,(-f_x, -f_y, 1)\, dxdy$$
$$dS = \sqrt{1 + f_x^2 + f_y^2}\, dxdy$$

For a level surface $g(x, y, z) = 0$, $g_z \neq 0$,

$$\hat{n}dS = \pm\,\frac{1}{g_z}\nabla g\, dxdy$$
$$dS = \frac{|\nabla g|}{|g_z|} dxdy$$

(Note: If $z = z(x, y)$ solves $g(x, y, z) = 0$, then $z_x = -g_x/g_z$ and $z_y = -g_y/g_z$)

Vector Identities

$$a \bullet (b \times c) = (a \times b) \bullet c = \text{determinant}(a, b, c)$$
$$a \times (b \times c) = (a \bullet c)b - (a \bullet b)c$$
$$\text{div}(\text{curl}\, \mathbf{F}) = \mathbf{0}$$
$$\text{curl}(\nabla f) = 0$$
$$\text{curl} = \sum_{n=1}^{3} \mathbf{i}_n \times \frac{\partial}{\partial x_n}$$
$$\nabla \bullet (f\mathbf{G}) = \nabla f \bullet \mathbf{G} + f\nabla \bullet \mathbf{G}$$
$$\nabla \times (f\mathbf{G}) = \nabla f \times \mathbf{G} + f\nabla \times \mathbf{G}$$
$$\nabla \bullet (\mathbf{F} \times \mathbf{G}) = (\nabla \times \mathbf{F}) \bullet \mathbf{G} - \mathbf{F} \bullet (\nabla \times \mathbf{G})$$
$$\nabla \times (\mathbf{F} \times \mathbf{G}) = (\nabla \bullet \mathbf{G} + \mathbf{G} \bullet \nabla)\mathbf{F} - (\nabla \bullet \mathbf{F} + \mathbf{F} \bullet \nabla)\mathbf{G}$$

Integral identities of vector fields

Green's theorem

$$\iint_S \text{div}\, \mathbf{F} dA = \int_{\partial S} \mathbf{F} \bullet \hat{n} ds$$
$$\iint_S \begin{vmatrix} \partial_x & \partial_y \\ G_1 & G_2 \end{vmatrix} dA = \int_{\partial S} \mathbf{G} \bullet d\mathbf{r}$$

Divergence theorem

$$\iiint_V \nabla \bullet \mathbf{F}\, dV = \iint_{\partial V} \mathbf{F} \bullet \hat{n}\, dS$$
$$\iiint_V \nabla \phi\, dV = \iint_{\partial V} \phi \hat{n}\, dS$$
$$\iiint_V \nabla \times \mathbf{G}\, dV = \iint_{\partial V} -\mathbf{G} \times \hat{n}\, dS$$

Stokes theorem

$$\iint_S \nabla \times \mathbf{F}\, dS = \int_{\partial S} \mathbf{F} \bullet d\mathbf{r}$$