1. Consider the curve parameterized by the vector function \( \mathbf{r}(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + \ln(\cos t) \mathbf{k} \) where \(-\pi/2 \leq t \leq \pi/2\).

(a) Find the curvature \( \kappa \) at the point \((\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \ln(\frac{1}{\sqrt{2}}))\).

(b) Find the principal normal vector \( \mathbf{N} \) at the point \((\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \ln(\frac{1}{\sqrt{2}}))\).

(c) Find the coordinates of the center of the osculating circle to the curve at the point \((\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \ln(\frac{1}{\sqrt{2}}))\).

2. Which of the following vector valued functions parameterize the semi-circle \( x^2 + y^2 = 1, \ y \geq 0 \) oriented in the clockwise direction?

A. \( \mathbf{r}(t) = (-\sin t, -\cos t) \) for \( \pi/2 \leq t \leq 3\pi/2 \).

B. \( \mathbf{r}(t) = (\sin t, \cos t) \) for \( 0 \leq t \leq \pi \).

C. \( \mathbf{r}(t) = (\sin^2 t - \cos^2 t, 2\sin t \cos t) \) for \( 0 \leq t \leq \pi/2 \).

D. \( \mathbf{r}(t) = (t^3, \sqrt{1-t^6}) \) for \( -1 \leq t \leq 1 \).

E. \( \mathbf{r}(t) = (t^2, \sqrt{1-t^4}) \) for \( -1 \leq t \leq 1 \).

F. \( \mathbf{r}(t) = (\frac{2t}{1+t^2}, \frac{1-t^2}{1+t^2}) \) for \( -1 \leq t \leq 1 \).

3. In this problem you may use the fact that \( \int \frac{dt}{\sqrt{1-t^2}} = \arcsin(t) + C \). Consider the curve with the parameterization

\[ \mathbf{r}(t) = 3t \mathbf{i} + 5\sqrt{1-t^2} \mathbf{j} + 4t \mathbf{k} \]

(a) Find the length of the curve from \( t = 0 \) to \( t = T \).

(b) Reparameterize the curve with respect to arclength measured from the point \((0, 5, 0)\) in the direction of increasing \( t \).

4. Let \( \mathbf{r}(t) \) be the position vector of a particle in motion. Let \( \mathbf{T}, \mathbf{N}, v, \kappa \) be the unit tangent vector, the principal unit normal vector, the speed, and the curvature respectively. Find an expression for \( \frac{d\mathbf{N}}{dt} \cdot \mathbf{T} \) in terms of \( \kappa \) and \( v \). in terms of \( \kappa \) and \( v \). (Hint: differentiate \( \mathbf{N} \cdot \mathbf{T} \).)