Other Old Math 317 Final Exam Questions

1) Consider the vector field

$$\vec{\mathbf{F}}(x,y,z) = (2xyz, x^2z, x^2y + 2z)$$

describing a force.

a) Show that $\vec{\mathbf{F}}$ is conservative.

b) Find the work done by $\vec{\mathbf{F}}$ in moving an object along the line segment beginning at $(1,1,1)$ and ending at $(2,2,4)$.

2) Consider the vector field

$$\vec{\mathbf{V}}(x,y) = (e^x \cos y + x^2, x^2y + 3)$$

Evaluate the line integral of $\vec{\mathbf{V}}$ around the oriented curve C obtained by moving from $(0,0)$ to $(1,0)$ to $(1,\pi)$ and finally to $(0,\pi)$ along straight line segments.

3) Evaluate

$$\iint_S xz\ dS$$

where S is the portion of the plane $x - 2y + z = 1$ inside the cylinder $x^2 + y^2 = 2$.

4) Consider the vector field

$$\vec{\mathbf{V}}(x,y,z) = (x + y, 2x - z, y^2 + z)$$

Evaluate the line integral of $\vec{\mathbf{V}}$ along the oriented curve C obtained by walking from $(2,0,0)$ to $(0,3,0)$ to $(0,0,6)$ and back to $(2,0,0)$ along straight line segments.

5) The sides of a grain silo are described by the portion of the cylinder $x^2 + y^2 = 1$ where $0 \le z \le 1$. The top of the silo is given by the portion of the sphere $x^2 + y^2 + z^2 = 2$ lying within the cylinder and above the xy–plane. Find the flux of the vector field

$$\vec{\mathbf{V}}(x,y,z) = (x^2yz, yz + e^x z, x^2 + y)$$

out of the silo.

6) Let C be the curve of intersection of the parabolic cylinder $x = y^2$ and the hyperbolic paraboloid $3z = 2xy$.

a) Write a vector parametric equation for C using x as the parameter.

b) Find the length of the part of C between the origin and the point $(9,3,18)$.

c) A particle moves along C in the direction for which x is increasing. If the particle moves with constant speed 9, find its velocity and acceleration vectors when it is at the point $(1,1,\frac{2}{3})$.

7) Let $\vec{\mathbf{F}} = 6x^2yz^2\hat{\boldsymbol{\imath}} + (2x^3z^2 + 2y - xz)\hat{\boldsymbol{\jmath}} + 4x^3yz\hat{\mathbf{k}}$ and let $\vec{\mathbf{G}} = yz\hat{\boldsymbol{\imath}} + xy\hat{\mathbf{k}}$.

a) For what value of the constant λ is the vector field $\vec{\mathbf{H}} = \vec{\mathbf{F}} + \lambda\vec{\mathbf{G}}$ conservative on 3-space?

b) Find a scalar potential $\phi(x,y,z)$ for the conservative field $\vec{\mathbf{H}}$ referred to in part (a).

c) Find $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ if C is the curve of intersection of the two surfaces $z = x$ and $y = e^{xz}$ from the point $(0,1,0)$ to the point $(1,e,1)$.

8) Let S be the part of the surface $z = xy$ lying inside the cylinder $x^2 + y^2 = 3$. Find the moment of inertia of S about the z-axis, that is,

$$I = \iint_S (x^2 + y^2)\ dS$$

9) Evaluate $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ where $\vec{\mathbf{F}} = x^2y^2\hat{\boldsymbol{\imath}} + 2xy\hat{\boldsymbol{\jmath}}$ and C is the boundary of the square in the xy–plane having one vertex at the origin and diagonally opposite vertex at the point $(3,3)$, oriented counterclockwise.

10) Let $\vec{\mathbf{F}} = -ye^z\hat{\boldsymbol{\imath}} + x^3\cos z\hat{\boldsymbol{\jmath}} + z\sin(xy)\hat{\mathbf{k}}$, and let S be the part of the surface $z = (1 - x^2)(1 - y^2)$ that lies above the square $-1 \le x \le 1, -1 \le y \le 1$ in the xy-plane. Find the flux of $\vec{\nabla} \times \vec{\mathbf{F}}$ upward through S.

11) Let B be the ball of volume V centered at the point (x_0, y_0, z_0), and let S be the sphere that is the boundary of B. Find the flux of $\vec{\mathbf{F}} = x^2\hat{\imath} + xy\hat{\jmath} + (3z - yz)\hat{\mathbf{k}}$ outward (from B) through S.

12 a) Explain why the integral form of Coulomb's law, $\iint_{\partial\Omega} \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dS = 4\pi \iiint_{\Omega} \rho\, dV$ implies its differential form $\vec{\nabla} \cdot \vec{\mathbf{E}} = 4\pi\rho$.

b) Explain why the integral form of Faraday's law, $\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = -\frac{1}{c} \iint_S \frac{\partial \vec{\mathbf{H}}}{\partial t}\, dS$, implies its differential form $\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{1}{c} \frac{\partial \vec{\mathbf{H}}}{\partial t}$

13) Let $\vec{\mathbf{F}} = (axy + z, x^2, bx + 2z)$.

a) For what values of a and b is $\vec{\mathbf{F}}$ conservative?

b) Taking the values of a and b from part (a), find a potential for $\vec{\mathbf{F}}$.

c) Taking the values of a and b from part (a), compute the integral $\int_c \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$,where C is the curve from $(2, 0, 1)$ to $(0, 2, 5)$ which goes along the intersection (in the first octant $x, y, z \geq 0$) of the paraboloid $z = (x - 1)^2 + y^2$ and the plane $z = 52x$.

14) Evaluate the integral $\iint_S \vec{\mathbf{F}} \cdot \hat{\mathbf{n}}\, dS$, where $\vec{\mathbf{F}} = (x, y, 1)$, and S is the surface $z = 1 - x^2 - y^2$ for $x^2 + y^2 \leq 1$ by two methods.

a) First, by a direct computation of the surface integral.

b) Second, by using the divergence theorem.

15) Evaluate the integral $\oint_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, in which $\vec{\mathbf{F}} = (e^{x^2} - yz, \sin y - yz, xz + 2y)$ and C is the triangular path from $(1, 0, 0)$ to $(0, 1, 0)$ to $(0, 0, 1)$ to $(1, 0, 0)$.