MATHEMATICS 317 April 1999 Final Exam

1. Let C be the curve of intersection of the surfaces $y = x^2$ and $z = \frac{2}{3}x^3$. A particle moves along C with constant speed such that $\frac{\mathrm{d}x}{\mathrm{d}t} > 0$. The particle is at $(0,0,0)$ at time $t = 0$ and is at $(3,9,18)$ at time $t = \frac{7}{2}$.

   (a) Find the length of the part of C between $(0,0,0)$ and $(3,9,18)$.

   (b) Find the constant speed of the particle.

   (c) Find the velocity of the particle when it is at $\left(1,1,\frac{2}{3}\right)$.

   (d) Find the acceleration of the particle when it is at $\left(1,1,\frac{2}{3}\right)$.

2. Let $\mathbf{F} = \left(y^2e^{3z} + Axy^3\right)\hat{\boldsymbol{\imath}} + \left(2xye^{3z} + 3x^2y^2\right)\hat{\boldsymbol{\jmath}} + Bxy^2e^{3z}\,\hat{\mathbf{k}}$.

   (a) For what values of the constants A and B is the vector field $\mathbf{F}$ conservative on $\mathbb{R}^3$?

   (b) If A and B have the values found in (a), find a scalar potential function for $\mathbf{F}$.

   (c) Let C be the curve $\mathbf{r} = e^{2t}\,\hat{\boldsymbol{\imath}} + e^{-t}\,\hat{\boldsymbol{\jmath}} + \ln(1+t)\,\hat{\mathbf{k}}$ from $(1,1,0)$ to $(e^2, \frac{1}{e}, \ln 2)$. Evaluate

$$\int_C \left(y^2e^{3z} + xy^3\right)\mathrm{d}x + \left(2xye^{3z} + 3x^2y^2\right)\mathrm{d}y + 3xy^2e^{3z}\,\mathrm{d}z$$

3. Let $\mathcal{S}$ be the part of the surface $z = xy$ that lies above the square $0 \le x \le 1$, $0 \le y \le 1$ in the xy–plane.

   (a) Find $\displaystyle\iint_{\mathcal{S}} \frac{x^2y}{\sqrt{1+x^2+y^2}}\,\mathrm{d}S$.

   (b) Find the flux of $\mathbf{F} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} + \hat{\mathbf{k}}$ upward through $\mathcal{S}$.

4. Evaluate $I = \displaystyle\oint_{\mathcal{C}} \left(\frac{1}{3}x^2y^3 - x^4y\right)\mathrm{d}x + \left(xy^4 + x^3y^2\right)\mathrm{d}y$ counterclockwise around the boundary of the half–disk $0 \le y \le \sqrt{4-x^2}$.

5. Find the flux of $\mathbf{F} = xy^2\hat{\boldsymbol{\imath}} + x^2y\hat{\boldsymbol{\jmath}} + \hat{\mathbf{k}}$ outward through the hemispherical surface

$$x^2 + y^2 + z^2 = 4, \qquad z \ge 0$$

6. Let C be a circle of radius R lying in the plane $x + y + z = 3$. Use Stokes' Theorem to calculate the value of

$$\oint_C \mathbf{F} \cdot d\mathbf{r}$$

   where $\mathbf{F} = z^2\hat{\boldsymbol{\imath}} + x^2\hat{\boldsymbol{\jmath}} + y^2\hat{\mathbf{k}}$. (You may use either orientation of the circle.)