

MATHEMATICS 317 April 1999 Final Exam

- [22] 1) Let C be the curve of intersection of the surfaces $y = x^2$ and $z = \frac{2}{3}x^3$. A particle moves along C with constant speed such that $\frac{dx}{dt} > 0$. The particle is at $(0, 0, 0)$ at time $t = 0$ and is at $(3, 9, 18)$ at time $t = \frac{7}{2}$.
- Find the length of the part of C between $(0, 0, 0)$ and $(3, 9, 18)$.
 - Find the constant speed of the particle.
 - Find the velocity of the particle when it is at $(1, 1, \frac{2}{3})$.
 - Find the acceleration of the particle when it is at $(1, 1, \frac{2}{3})$.
- [18] 2) Let $\vec{\mathbf{F}} = (y^2e^{3z} + Axy^3)\hat{\mathbf{i}} + (2xye^{3z} + 3x^2y^2)\hat{\mathbf{j}} + Bxy^2e^{3z}\hat{\mathbf{k}}$.
- For what values of the constants A and B is the vector field $\vec{\mathbf{F}}$ conservative on \mathbb{R}^3 ?
 - If A and B have the values found in (a), find a scalar potential function for $\vec{\mathbf{F}}$.
 - Let C be the curve $\vec{\mathbf{r}} = e^{2t}\hat{\mathbf{i}} + e^{-t}\hat{\mathbf{j}} + \ln(1+t)\hat{\mathbf{k}}$ from $(1, 1, 0)$ to $(e^2, \frac{1}{e}, \ln 2)$. Evaluate

$$\int_C (y^2e^{3z} + xy^3) dx + (2xye^{3z} + 3x^2y^2) dy + 3xy^2e^{3z} dz$$

- [15] 3) Let S be the part of the surface $z = xy$ that lies above the square $0 \leq x \leq 1, 0 \leq y \leq 1$ in the xy -plane.
- Find $\iint_S \frac{x^2y}{\sqrt{1+x^2+y^2}} dS$.
 - Find the flux of $\vec{\mathbf{F}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + \hat{\mathbf{k}}$ upward through S .
- [15] 4) Evaluate $I = \oint_C (\frac{1}{3}x^2y^3 - x^4y) dx + (xy^4 + x^3y^2) dy$ counterclockwise around the boundary of the half-disk $0 \leq y \leq \sqrt{4-x^2}$.
- [15] 5) Find the flux of $\vec{\mathbf{F}} = xy^2\hat{\mathbf{i}} + x^2y\hat{\mathbf{j}} + \hat{\mathbf{k}}$ outward through the hemispherical surface $x^2 + y^2 + z^2 = 4, z \geq 0$. (Hint: the flux can be calculated directly, but it is rather easier to calculate it using the Divergence Theorem.)
- [15] 6) Let C be a circle of radius R lying in the plane $x + y + z = 3$. Use Stokes' Theorem to calculate the value of

$$\oint_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$$

where $\vec{\mathbf{F}} = z^2\hat{\mathbf{i}} + x^2\hat{\mathbf{j}} + y^2\hat{\mathbf{k}}$. (You may use either orientation of the circle.)