1. Let 
\[ \mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}, \quad r = |\mathbf{r}| \]

(a) Compute \( a \) where \( \nabla \left( \frac{1}{r} \right) = -r^2 \mathbf{r} \).
(b) Compute \( a \) where \( \nabla \cdot (r \mathbf{r}) = ar \).
(c) Compute \( a \) where \( \nabla \cdot (\nabla (r^3)) = ar \).

2. Let \( \mathbf{r}(t) = \left( \frac{1}{3} t^3, \frac{1}{2} t^2, \frac{1}{2} t \right), t \geq 0 \). Compute \( s(t) \), the arclength of the curve at time \( t \).

3. A particle of mass \( m = 2 \) is acted on by a force
\[ \mathbf{F} = (4t, 6t^2, -4t) \]
At \( t = 0 \), the particle has velocity zero and is located at the point \( (1, 2, 3) \).

(a) Find the velocity vector \( \mathbf{v}(t) \) for \( t \geq 0 \).
(b) Find the position vector \( \mathbf{r}(t) \) for \( t \geq 0 \).
(c) Find \( \kappa(t) \) the curvature of the path traversed by the particle for \( t \geq 0 \).
(d) Find the work done by the force on the particle from \( t = 0 \) to \( t = T \).

4. Let
\[ \mathbf{F} = \left( \frac{2z}{1+y} + \sin(x^2), \frac{3z}{1+x} + \sin(y^2), 5(x+1)(y+2) \right) \]
Let \( C \) be the oriented curve consisting of four line segments from \( (0,0,0) \) to \( (2,0,0) \), from \( (2,0,0) \) to \( (0,0,2) \), from \( (0,0,2) \) to \( (0,3,0) \), and from \( (0,3,0) \) to \( (0,0,0) \).

(a) Draw a picture of \( C \). Clearly indicate the orientation on each line segment.
(b) Compute the work integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \).

5. Recall that if \( \mathbf{T} \) is the unit tangent vector to an oriented curve with arclength parameter \( s \), then the curvature \( \kappa \) and the principle normal vector \( \mathbf{N} \) are defined by the equation
\[ \frac{d\mathbf{T}}{ds} = \kappa \mathbf{N} \]
Moreover, the torsion \( \tau \) and the binormal vector \( \mathbf{B} \) are defined by the equations
\[ \mathbf{B} = \mathbf{T} \times \mathbf{N}, \quad \frac{d\mathbf{B}}{ds} = -\tau \mathbf{N} \]
Show that
\[ \frac{d\mathbf{N}}{ds} = -\kappa \mathbf{T} + \tau \mathbf{B} \]
6. Let \( a, b, c \) be constants and let 
\[
F = (2bx^2y^2 - axz, 2ax^3y - 3cz^2, 2byz + x^2)
\]

(a) Compute \( \nabla \times F \).
(b) Find the values of \( a, b, c \) which make \( F \) conservative.
(c) Using the values of \( a, b, c \) from the previous part, find a potential function for \( F \).
(d) Using the values of \( a, b, c \) from the previous part, evaluate \( \int_C F \cdot dr \) where \( \mathbf{r}(t) = (\cos(t), 2\sin(t), \frac{2t}{\pi} + 1) \) for \( 0 \leq t \leq \frac{\pi}{2} \).

7. (a) Evaluate
\[
\int_C \sqrt{1 + x^3} \, dx + (2xy^2 + y^2) \, dy
\]
where \( C \) is the unit circle \( x^2 + y^2 = 1 \), oriented counterclockwise.
(b) Evaluate
\[
\int_C \sqrt{1 + x^3} \, dx + (2xy^2 + y^2) \, dy
\]
where \( C \) is now the part of the unit circle \( x^2 + y^2 = 1 \), with \( x \geq 0 \), still oriented counterclockwise.

8. Let \( S \) be the part of the sphere \( x^2 + y^2 + z^2 = 2 \) where \( y \geq 1 \), oriented away from the origin.

(a) Compute
\[
\iint_S y^3 \, dS
\]
(b) Compute
\[
\iint_S (xy \mathbf{i} + xz \mathbf{j} + zy \mathbf{k}) \cdot \mathbf{n} \, dS
\]

9. Let \( S \) be the sphere \( x^2 + y^2 + z^2 = 3 \) oriented inward. Compute the flux integral
\[
\iint_S F \cdot \mathbf{n} \, dS
\]
where
\[
F = (xy^2 + y^4z^6, yz^2 + x^4z, zx^2 + xy^4)
\]