MATHEMATICS 317 December 2006 Final Exam

1. Evaluate the integral

$$\int_C xy\,\mathrm{d}x + yz\,\mathrm{d}y + zx\,\mathrm{d}z$$

around the triangle with vertices $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$, oriented clockwise as seen from the point $(1,1,1)$.

2. Let C be the curve in the xy–plane from the point $(0,0)$ to the point $(5,5)$ consisting of the ten line segments consecutively connecting the points $(0,0)$, $(0,1)$, $(1,1)$, $(1,2)$, $(2,2)$, $(2,3)$, $(3,3)$, $(3,4)$, $(4,4)$, $(4,5)$, $(5,5)$. Evaluate the line integral

$$\int_C \mathbf{F}\cdot\mathrm{d}\mathbf{r}$$

where

$$\mathbf{F} = y\,\hat{\boldsymbol{\imath}} + (2x-10)\,\hat{\boldsymbol{\jmath}}$$

3. Let S be the surface given by the equation

$$x^2 + z^2 = \sin^2 y$$

lying between the planes $y=0$ and $y=\pi$. Evaluate the integral

$$\iint_S \sqrt{1+\cos^2 y}\,\mathrm{d}S$$

4. Let S be the part of the sphere $x^2+y^2+z^2=4$ between the planes $z=1$ and $z=0$ oriented away from the origin. Let

$$\mathbf{F} = (e^y + xz)\,\hat{\boldsymbol{\imath}} + (zy + \tan(x))\,\hat{\boldsymbol{\jmath}} + (z^2-1)\,\hat{\mathbf{k}}$$

Compute the flux integral

$$\iint_S \mathbf{F}\cdot\hat{\mathbf{n}}\,\mathrm{d}S.$$

5. Let

$$\mathbf{r}(t) = \cos^3 t\,\hat{\boldsymbol{\imath}} + \sin^3 t\,\hat{\boldsymbol{\jmath}} + \frac{3}{2}\sin t\cos t\,\hat{\mathbf{k}}$$

Reparameterize $\mathbf{r}(t)$ with respect to arclength measured from the point $t=0$ in the direction of increasing t.

6. Let

$$\mathbf{r}(t) = t^2\,\hat{\boldsymbol{\imath}} + 2t\,\hat{\boldsymbol{\jmath}} + \ln t\,\hat{\mathbf{k}}$$

Compute the unit tangent and unit normal vectors $\hat{\mathbf{T}}(t)$ and $\hat{\mathbf{N}}(t)$. Compute the curvature $\kappa(t)$. Simplify whenever possible!

7. Show that the following line integral is independent of path and evaluate the integral.

$$\int_C (ye^x + \sin y)\,\mathrm{d}x + (e^x + \sin y + x\cos y)\,\mathrm{d}y$$

where C is any path from $(1,0)$ to $(0,\pi/2)$.

8. Let

$$\mathbf{F} = \frac{-z}{x^2+z^2}\,\hat{\boldsymbol{\imath}} + y\,\hat{\boldsymbol{\jmath}} + \frac{x}{x^2+z^2}\,\hat{\mathbf{k}}$$

(a) Determine the domain of $\mathbf{F}$.

(b) Determine the curl of $\mathbf{F}$. Simplify if possible.

(c) Determine the divergence of $\mathbf{F}$. Simplify if possible.

(d) Is $\mathbf{F}$ conservative? Give a reason for your answer.