MATHEMATICS 317 December 2013 Final Exam Answers

1. (a) $2x + y + 3z = 6$ (b) $\kappa(t) = \frac{2\sqrt{1+9t^2+9t^4}}{[1+4t^2+9t^4]^{3/2}}$

2. (a) $\hat{\mathbf{T}}(1) = \frac{(2,0,1)}{\sqrt{5}}$ (b) $\frac{1}{3}\left[5^{3/2} - 8\right]$

3. (a) $\boldsymbol{\nabla} \times \mathbf{F} = \mathbf{0}$. $\mathbf{F}$ is conservative. (b) $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r} = 2\pi^2$

4. (a) 8 (b) $4\sqrt{3}$

5. (a) $\mathbf{r}(\theta, z) = \frac{2}{3}(3-z)\cos\theta\,\hat{\boldsymbol{\imath}} + \frac{2}{3}(3-z)\sin\theta\,\hat{\boldsymbol{\jmath}} + z\,\hat{\mathbf{k}}$ $\quad 0 \le \theta < 2\pi, \quad 0 \le z \le 3.$

   (b) 1

6. (a) $\frac{1}{\sqrt{3}}(-1,-1,1)$ (b) $-\frac{27\pi}{2}$ (c) $-\frac{81\pi}{2}$

7. (a) $\boldsymbol{\nabla} \cdot \mathbf{F} = 2 + 2z$ (b) $\pi\frac{23}{6}5^3 = 479\frac{1}{6}\pi$

   (c) Let S be an oriented surface that encloses a solid V and has outward pointing normal. If $\bar{z} = -\frac{9}{2|V|} - 1$, where $|V|$ is the volume of V and $\bar{z}$ is the z–component of the centroid (i.e. centre of mass with constant density) of V, then $\iint_S \mathbf{F} \cdot \hat{\mathbf{n}}\,\mathrm{d}S = -9$. One surface which obeys this condition is the unit cube (with outward normal) centred on $\left(0, 0, -\frac{11}{2}\right)$.

8. (a) True (b) True (c) True (d) False (e) True

   (f) That depends. If $\kappa = 0$, the curve is part of a straight line. If $\kappa > 0$ it is part of a circle of radius $\frac{1}{\kappa}$.

   (g) False (h) False (i) False