MATHEMATICS 317 December 2010 Final Exam Answers

1. (a) \( \mathbf{r}'(t) = (-3 \sin t, 3 \cos t, 4) \)  
   (b) 5

2. \(-\frac{2}{3}\)

3. (a) \( y = 2x - 2 \)  
   (b) parabolic bowl  
   (c) positive y direction

4. \(-\frac{1}{3}\)

5. (a) \( D = \{ (x, y, z) \mid x > 0, y > 0, z > 0 \} \)  
   (b) The domain \( D \) is both connected and simply connected.  
   (c) \( \nabla \times \mathbf{F} = (2x - \frac{1}{x})\mathbf{\hat{k}} \)  
   (d) \( 2 \ln 2 - 24 \)  
   (e) No. \( \mathbf{F} \) is not conservative.

6. \(-32\pi\)

7. \(-\pi\)

8. See the solution.

9. (a) decreasing  
   (b) \( f(x) \) is \( D \)  
   (c) \( \mathbf{r}(\theta) = \cos \theta \mathbf{\hat{i}} + \sin \theta \mathbf{\hat{k}} + \sin \theta \cos \theta \mathbf{\hat{j}}, \ 0 \leq \theta < 2\pi \)  
   (d) We want parametrisation (d) with domain \( |u| \geq 2, 0 \leq v \leq 5 \).  
   (e) One possible answer is \( \mathbf{r}(t) = t\mathbf{\hat{i}}, 0 \leq t \leq 1 \).  
   (f) \( C = 6 \)  
   (g) \( \{ (a, b, c, d) \mid a, b, c, d \text{ all real and } b = c \} \)  
   (h) 2  
   (i) (1) True  
   (2) False  
   (3) False  
   (4) False  
   (5) False
   (j) Any vector field whose divergence is 1 everywhere will work. One such vector field is \( \mathbf{F} = x \mathbf{\hat{i}} \).  
   (k) negative

10. (a) false  
    (b) false  
    (c) true  
    (d) false  
    (e) true, assuming that the second derivatives of the vector field exist and are continuous.  
    (f) silly, but true  
    (g) true  
    (h) false  
    (i) false  
    (j) false