MATHEMATICS 317 December 2000 Final Exam Answers

1. (a) $\hat{\mathbf{T}} = \frac{1}{\sqrt{6}}\big(0, 2, -\sqrt{2}\big)$, $\hat{\mathbf{B}} = \frac{1}{\sqrt{13}}\big(-1, 2, 2\sqrt{2}\big)$, $\hat{\mathbf{N}} = -\frac{1}{\sqrt{39}}\big(6, 1, \sqrt{2}\big)$, $\kappa = \frac{\sqrt{39}}{9}$
   (b) $\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{5}{3}\sqrt{6}$, $\mathbf{v} = (0, \sqrt{2}, -1)$
2. (a) $p = 2$, $m = 2$, and $n = 2$, but $q \in \mathbb{R}$ is completely free (b) $4q$
3. (a) $I = \frac{\sqrt{a^2+b^2}}{a} \iint_{\mathcal{D}} (x^2 + y^2)\,\mathrm{d}x\mathrm{d}y$ (b) $I = \iint_{\mathcal{R}} \big(t^2\big) t\sqrt{1 + b^2/a^2}\,\mathrm{d}t\,\mathrm{d}\theta$
   (c) $I = \frac{\pi}{2} a^3 \sqrt{a^2 + b^2}$
4. 24π
5. (a) $\iint_{\mathcal{S}} \mathbf{F} \cdot \hat{\mathbf{n}}\,\mathrm{d}S = 0$ (b) $\iint_{\mathcal{S}_{\text{side}}} \mathbf{F} \cdot \hat{\mathbf{n}}\mathrm{d}S = \frac{15}{2}\pi$
6. (a) $\kappa = 1$ (b) $\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{r} \times \mathbf{v} = \mathbf{0}$ (c) $\iint_{\mathcal{S}} (x\,\hat{\boldsymbol{\imath}} - y\,\hat{\boldsymbol{\jmath}} + z^2\,\hat{\mathbf{k}}) \cdot \hat{\mathbf{n}}\mathrm{d}S = 8\pi b^2$
7. (a), (b) See the solution.