

## Math 302/Stat 302 Problem Set IX

Due March 29

### Problems from textbook

page 275 # 6.1, 6.3, 6.6, 6.7

### Other Problems

I) The concentration of population in cities seems to obey the law

$$C(r) = ae^{-br}, \quad r \geq 0$$

as a function of the distance  $r$  from the centre. Here  $a$  and  $b$  are positive constants.

- Find the probability density function for  $r$ .
- Find the average distance of a resident from the centre.
- (Optional.) Find the smallest radius necessary in a public transportation network that will serve 80% of the residents.
- (Optional.) Find the average distance from the centre of a resident served by the transportation network.

II) The epicentre of an earthquake is equally likely to be at any point along a fault, which is a straight line. The distance between a dam and the fault is  $w$ . Find the probability density function for  $R$ , the distance between the epicentre and the dam. Restrict  $R \leq r_0$ . (Earthquakes at distances greater than  $r_0$  have negligible effect.)

III) Suppose that  $X$  and  $Y$  are random variables with joint probability density

$$f(x, y) = \begin{cases} \frac{4}{5}(xy + 1) & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- Are  $X$  and  $Y$  independent?
- Compute the marginal densities  $f_1(x)$ ,  $f_2(y)$  and the conditional density  $f_{X|Y}(x|y)$ .
- Calculate the means  $\mu_X$ ,  $\mu_Y$ , the variances  $\sigma_X^2$ ,  $\sigma_Y^2$  and the covariance  $\sigma_{XY}$ .
- Calculate  $E(X^2 + Y^2)$ .

IV)  $Y_1$  and  $Y_2$  have a joint probability density function given by

$$f(y_1, y_2) = \begin{cases} \frac{1}{2} & \text{if } 0 \leq y_2 \leq y_1 \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- Compute the marginal probability density functions for  $Y_1$  and  $Y_2$ .
- Compute  $P(Y_1 \leq 1, Y_2 \leq \frac{1}{2})$ .

V) Let  $R$  be the region between  $y = x$  and  $y = x^2$ . A random point  $(X, Y)$  is selected from  $R$ . Find the joint probability density function of  $X$  and  $Y$ .

**Exam is Thursday, April 13 at 8:30 in WOOD 2.**