Computation of the Poisson Distribution

$$P(Y=y) = \lim_{n\to\infty} \binom{n}{y} \left(\frac{\lambda}{n}\right)^{y} \left(1-\frac{\lambda}{n}\right)^{n-y}$$

$$= \lim_{n\to\infty} \frac{n!}{y!(n-y)!} \left(\frac{\lambda}{n}\right)^{y} \left(1-\frac{\lambda}{n}\right)^{n-y}$$

$$= \lim_{n\to\infty} \frac{\lambda^{y}}{y!} \frac{n \quad (n-1) \quad \cdots \quad (n-y+1)}{n \quad n \quad \cdots \quad n} \left(1-\frac{\lambda}{n}\right)^{n-y}$$

$$= \lim_{n\to\infty} \frac{\lambda^{y}}{y!} \frac{n \quad (n-1) \quad \cdots \quad (n-y+1)}{n \quad n \quad \cdots \quad n} \left(1-\frac{\lambda}{n}\right)^{-y} \left(1-\frac{\lambda}{n}\right)^{n}$$

$$= \lim_{n\to\infty} \frac{\lambda^{y}}{y!} \frac{1}{1-\frac{\lambda}{n}} \frac{1-\frac{1}{n}}{1-\frac{\lambda}{n}} \cdots \frac{1-\frac{y-1}{n}}{1-\frac{\lambda}{n}} \left(1-\frac{\lambda}{n}\right)^{n}$$

$$= \frac{\lambda^{y}}{y!} \lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^{n}$$

$$= e^{-\lambda}\frac{\lambda^{y}}{y!}$$