April 1999 MATH/STAT 302 Exam

Students writing the exam were instructed to do either "Part I" or "Part II" of each of questions 4 through 7.

1) A certain firm produces resistors and markets them as 10–ohm resistors. However the actual ohms of resistance produced by the resistors may vary. Research has established that 10% of the values are below 9.5 ohms and 20% are above 10.5 ohms. If two resistors, randomly selected, are used in a system, find the probabilities of the following events:
   a) Both resistors have actual values between 9.5 and 10.5 ohms.
   b) At least one resistor has an actual value in excess of 10.5 ohms.

2) Two assembly lines (I and II) have the same rate of defectives in their production of voltage regulators. Five regulators are sampled from each line and tested. Among the total of 10 tested regulators, 4 are defective. Find the probability that exactly 3 of the defectives came from line I.

3) Two sentries are sent to patrol a road that is 1 km long. The sentries are sent to points chosen independently and at random along the road. Find the probability that the sentries will be less than 1/4 kilometer apart when they reach their assigned posts.

4) **Part I**. A large construction firm has won 60% of the jobs for which it has bid. Suppose that this firm bids on 25 jobs next month. Approximate the probability that it will win at least 20 of these jobs.
   Part II. For a certain section of pine forest, the number Y of diseased trees per acre has a Poisson distribution with mean $\lambda = 10$. The diseased trees are sprayed with an insecticide at a cost of \$3 per tree, plus a fixed overhead cost for equipment rental of \$50. Letting C denote the total spraying cost for a randomly selected acre, find the expected value and standard deviation for C.

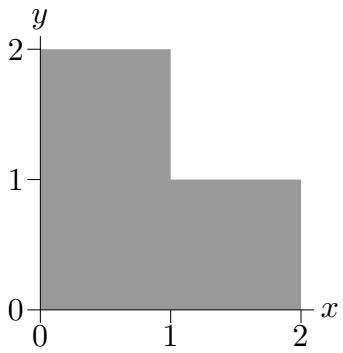
5) **Part I**. A dart lands at random coordinates (X, Y) on the square $[0, 1] \times [0, 1]$. Find the probability density of the area of the rectangle with corners $(0, 0)$ and (X, Y).
   Part II. A dart lands on a point (X, Y) that is uniformly distributed over the surface of a dart board with unit radius and "bull's eye" placed at the origin. [In other words, the dart board is the set of all (x, y) for which $x^2 + y^2 \leq 1$.] Let $R = \sqrt{X^2 + Y^2}$ denote the distance of the dart from the bull's eye (i.e. origin).
   a) Find the density function of R.
   b) Find $E(R)$.

6) **Part I**. An electronic surveillance system has one of each of two different components in joint operation. If X and Y denote the random life lengths of the components of type I and type II, respectively, the joint probability density function is given by $f(x,y) = k\,x\,e^{-(x+y)/2}$ if $x > 0$ and $y > 0$ and it is zero otherwise. Here $k = \frac{1}{8}$.

a) Are X and Y independent? [State why or why not.]

b) Find $P(X > 1, Y > 1)$.

Part II. Consider the joint density function $f(x,y)$ of X and Y which is constant on the shaded region of the following figure.

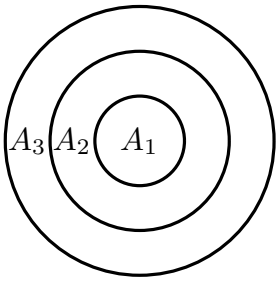


a) Find the distribution of X given $Y = y$.

b) Compute $E(X|Y = 3/2)$ and $\mathrm{Var}(X|Y = 3/2)$.

c) Suppose that you have to guess the value of X after being told the value of Y. You will win a larger amount if your guess is close to the truth. Would you rather face the situation where you are told (choose one) $Y = 1/2$ or $Y = 3/2$? Explain.

7) **Part I**. Five darts are thrown at the target below and land in one of the areas A_1, A_2 or A_3 with probabilities $p_1 = \frac{1}{6}$, $p_2 = \frac{1}{3}$, $p_3 = \frac{1}{2}$, respectively. Find the probability that three of the five darts land in area A_1 or A_2.



Part II. Three power board assemblers B_1, B_2 and B_3 produce power boards at varying rates of speed and at varying error rates. For example, in one day, B_1 assembles 25% of all of the boards produced and 2% of B_1's boards are defective. The following table gives a complete summary.

Assembler	B_1	B_2	B_3
% of boards produced	25%	45%	30%
% of boards defective	2%	3%	2%

A board is randomly drawn off of the assembly line and found to be defective. What is the probability that it was made by B_2?

8) A system with n parallel components will continue to function during the next 400 days as long as at least 1 component functions. The components operate independently and each will survive the next 400 days with probability $\frac{1}{200}$.
   a) How big must n be to insure a probability of at least $1 - \frac{1}{e}$ that the system survives the next 400 days.
   b) Repeat (a) using a convenient and valid approximation.

9) Suppose that X and Y have joint distribution given by $f(x, y) = x + y$ for $0 \leq x, y \leq 1$ and 0 otherwise. Find the distribution of $X + Y$.

10) a) A random variable X is uniformly distributed over the unit interval $(0, 1)$. Find the distribution of $-\ln X$, ln denoting the natural logarithm.
    b) Suppose that $X_1, \cdots, X_n$ are n such uniform variables (all independent). State what happens in precise mathematical terms to the quantity $G = \left(X_1 \times \cdots \times X_n\right)^{1/n}$ as n tends to ∞.