MATH 267 Problem Set 9

1. Find the 4×4 matrices (with complex entries) F_4 and G_4 that satisfy

$$\begin{bmatrix} \widehat{x}[0] \\ \widehat{x}[1] \\ \widehat{x}[2] \\ \widehat{x}[3] \end{bmatrix} = F_4 \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix} \qquad \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix} = G_4 \begin{bmatrix} \widehat{x}[0] \\ \widehat{x}[1] \\ \widehat{x}[2] \\ \widehat{x}[3] \end{bmatrix}$$

   for any periodic signal $x[\,\cdot\,]$ of period four with discrete Fourier transform $\widehat{x}[\,\cdot\,]$.

2. Each vector below is one period of a periodic discrete signal with the same name. Compute the Discrete Fourier Transform for each of these signals. Return the same number of components as you find in the input vector. Each vector starts with index 0.

$$a = [1, 1, 1, 1] \qquad b = [1, 1, \cdots, 1, 1] \text{ (N entries)}$$
$$c = [1, r, r^2, \cdots, r^{N-1}] \text{ (for r constant, $|r| \neq 1$)} \qquad d = [1, -1]$$
$$e = [1, 0, 1, 0]$$

3. Find the Inverse Discrete Fourier Transform for each vector below.

$$\widehat{a} = [1, 1, \cdots, 1, 1] \text{ (N entries)} \qquad \widehat{b} = [1, r, r^2, \cdots, r^{N-1}] \text{ (for r constant, $|r| \neq 1$)}$$
$$\widehat{c} = [0, 0, 1, 0] \qquad \widehat{d} = [4, 3, 2, 1]$$

4. Two N-periodic sequences x, y and their discrete Fourier transforms $\widehat{x}$, $\widehat{y}$ are given. Prove:

$$f[n] = e^{2\pi i \frac{kp}{N}} x[n] \implies \widehat{f}[k] = \widehat{x}[k-p] \quad \text{(frequency shifting)}$$
$$f[n] = \tfrac{1}{N} \sum_{m=0}^{N-1} x[m]\, y[n-m] \implies \widehat{f}[k] = \widehat{x}[k]\, \widehat{y}[k] \quad \text{(convolution)}$$
$$f[n] = x[n]\, y[n] \implies \widehat{f}[k] = \sum_{m=0}^{N-1} x[m]\, y[k-m] \quad \text{(multiplication)}$$