Math 267 Problem Set 11

The notation

$$u[n] = \left\{ \begin{array}{ll} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0 \end{array} \right\}, \qquad \delta[n] = \left\{ \begin{array}{ll} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0 \end{array} \right\}$$

will be used throughout this problem set.

1. The output $y[n]$ generated by a system S for the input $x[n]$ is

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k]$$

where $g[n] = u[n] - u[n-4]$.
(a) Determine $y[n]$ when $x[n] = \delta[n-1]$.
(b) Determine $y[n]$ when $x[n] = \delta[n-2]$.
(c) Is the system LTI?
(d) Determine $y[n]$ when $x[n] = u[n]$.

2. Determine the z–transforms for each of the following signals. Sketch the pole–zero plot and indicate the region of convergence. If the unit circle, $|z| = 1$, is in the ROC, find the signal's Fourier Transform as well.
(a) $\left(\frac{1}{2}\right)^n \{u[n+4] - u[n-5]\}$
(b) $(-2)^n u[n-3]$
(c) $\left(\frac{1}{2}\right)^{|n|}$
(d) $4^n \cos\left(\frac{2\pi}{6}n + \frac{\pi}{4}\right) u[-n-1]$

3. Let

$$x[n] = (-1)^n u[n] + \alpha^n u[-n-n_0]$$

Determine the constraints imposed on the complex number α and the integer n_0 by the requirement that the z–transform of $x[n]$ has region of convergence $1 < |z| < 2$.

4. Use partial fractions and the fact that the z–transform of $a^n u[n]$ is $\frac{1}{1-az^{-1}}$, with region of convergence $|z| > |a|$, to find the inverse z–transform of

$$X(z) = \frac{1 - \frac{1}{3}z^{-1}}{(1-z^{-1})(1+2z^{-1})}, \qquad |z| > 2$$