

Math 267 Problem Set 1

1. Solve the initial value problems **and sketch** the solutions:

(i)

$$y'' - 6y' + 9y = 0, \quad y(0) = 0, \quad y'(0) = 2.$$

(ii)

$$y'' + 4y = 3 \sin 2t, \quad y(0) = 2, \quad y'(0) = -1.$$

2. Find all solutions of the boundary value problems **and sketch** solutions (if they exist). If there's more than one solution, indicate which solution you are sketching:

(i)

$$y'' + 2y = 0, \quad y(0) = 1, \quad y'(\pi) = 0.$$

(ii)

$$y'' + y = 0, \quad y(0) = 0, \quad y(L) = 0.$$

(iii)

$$y'' + 4y = \sin x, \quad y(0) = 0, \quad y(\pi) = 0.$$

(iv)

$$y'' + y = x, \quad y(0) = 0, \quad y(1) = 0.$$

3. Find all real values of λ such that

$$y'' + \lambda y = 0, \quad y(0) = 0, \quad y(L) = 0$$

has a non-zero solution.

4. Find all real values of λ such that

$$y'' + \lambda y = 0, \quad y(0) = 0, \quad y'(L) = 0$$

has a non-zero solution.

5. Find all real values of λ such that

$$y'' - \lambda y = 0, \quad y(0) = 0, \quad y'(L) = 0$$

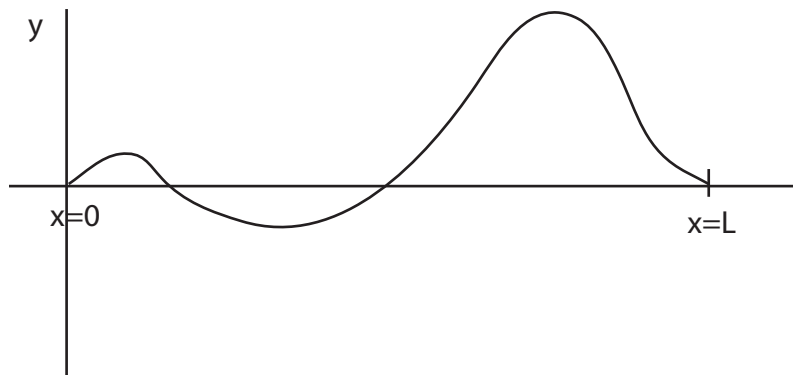
has a non-zero solution.

6. Find all real values of λ such that

$$y'' - \lambda y = 0, \quad y(-\pi) = y(\pi), \quad y'(-\pi) = y'(\pi)$$

has a non-zero solution.

7. In class we will derive the **wave equation** for an elastic string fixed at both ends.



$$\frac{\partial^2 y}{\partial t^2} - a^2 \frac{\partial^2 y}{\partial x^2} = 0 \quad y(0, t) = y(L, t) = 0.$$

The function that describes the shape of the string at time t is $y(x, t)$. The parameter $a = \sqrt{T/\rho}$ where T is the tension in the string and ρ is the density (mass per unit length) of the string. Let's suppose that $L = 1\text{m}$ for this problem.

(i) Show (by substituting in) that the function

$$y(x, t) = \sum_{n=1}^{\infty} c_n \sin(n\pi x) \cos(n\pi a t)$$

satisfies the equation and the boundary conditions (the c_n are numbers).

(ii) Now suppose that the initial position of the string is given by

$$y(x, 0) = \sin(2\pi x)$$

and that the string is initially motionless:

$$\frac{\partial y}{\partial t}(x, 0) = 0.$$

Guess coefficients c_n so that the $y(x, t)$ of part (i) satisfies these two initial conditions.

(iii) Sketch $y(x, 0)$, $y(x, 1/a)$ and $y(x, 2/a)$.

(iv) Finally, suppose that the density of the string is $\rho = .01\text{kg/m}$ and recall that the speed of sound in air is 340m/s . Find the minimum tension T that the string must be under in order that part of the string will reach the speed of sound.