Example of Newtons Method

$$f(x,y) = 1 + (x-2)^4 + (x-2)^2 y^2 + (y+1)^2$$

$$\nabla f(x,y) = \begin{bmatrix} f_x \\ f_y \end{bmatrix} = \begin{bmatrix} 4(x-2)^3 + 2(x-2)y^2 \\ 2(x-2)^2 y + 2(y+1) \end{bmatrix}$$

$$H(x,y) = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} = \begin{bmatrix} 12(x-2)^2 + 2y^2 & 4(x-2)y \\ 4(x-2)y & 2(x-2)^2 + 2 \end{bmatrix}$$

$$\vec{x}_{k+1} = \vec{x}_k - H(\vec{x}_k)^{-1} \nabla f(\vec{x}_k)$$

Initialization

$\vec{x}_0 = \langle 1, 1 \rangle$ MATLAB: $X = [1;1]$;

First Step

Math	MATLAB
$\nabla f(1,1) = \begin{bmatrix} -6 \\ 6 \end{bmatrix}$	G = gradf(X);
$H(1,1) = \begin{bmatrix} 14 & -4 \\ -4 & 4 \end{bmatrix}$	H = hessf(X);
$\vec{x}_1 - \vec{x}_0 = -H(1,1)^{-1} \nabla f(1,1) = \left\langle 0, -\frac{3}{2} \right\rangle$	dX = -H\G ;
$\vec{x}_1 = \langle 1, -0.5 \rangle$	X = X +dX ;

First Eight Steps

k	x_k	y_k
0	1.0000	1.0000
1	1.0000	-0.5000
2	1.3913	-0.6957
3	1.7459	-0.9488
4	1.9863	-1.0482
5	1.9987	-1.0002
6	2.0000	-1.0000
7	2.0000	-1.0000
8	2.0000	-1.0000