

Three Dimensional Determinants

The determinant of a 3×3 matrix can be defined in terms of some 2×2 determinants by

$$\det \begin{bmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} = i \det \begin{bmatrix} \cancel{j} & \cancel{k} \\ a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} - j \det \begin{bmatrix} \cancel{i} & \cancel{k} \\ a_1 & a_3 \\ b_1 & b_3 \end{bmatrix} + k \det \begin{bmatrix} \cancel{i} & \cancel{j} \\ a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}$$
$$= i(a_2b_3 - a_3b_2) - j(a_1b_3 - a_3b_1) + k(a_1b_2 - a_2b_1)$$

This formula is called “expansion along the top row”. There is one term in the formula for each entry in the top row. The term is a sign times the entry itself times the determinant of the 2×2 matrix gotten by deleting the row and column that contains the entry. The sign alternates, starting with a +.