December 1999 MATH 256 Exam

1) (a) Suppose that the area $A(t)$ of a disk is changing at a rate proportional to its radius $R(t)$. If $A(0) = 2$ and $A(1) = 1$, when will the disk disappear?
(b) Suppose that the rate of change of $A(t)$ at time t is equal to $\pi\sqrt{R(t)}$. If $A(0) = \pi$, at what time will $A(t) = 2$?

2) Suppose that the numerical calculation of the solution to an initial value problem $y' = f(y,t)$, $y(0) = 0$, using one of Euler, Improved Euler of Runge–Kutta methods yield the following results:

Step size h	Approximation for $y(1)$
0.50	1.6325
0.10	1.4947
0.05	1.4904

a) Which method was used? How do you know?
b) Given this data, what is your best estimate for $y(1)$?.

3) Find the general solution of

$$y'' + 9y = x\cos(3x)$$

4) Solve

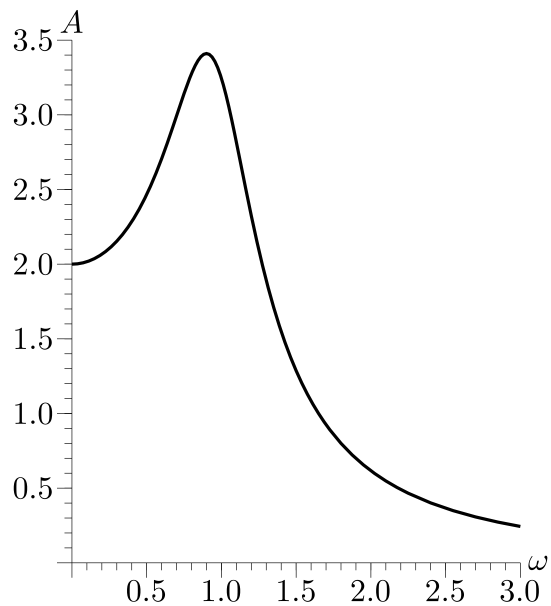
$$\vec{x}'(t) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \vec{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

with initial condition $\vec{x}(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

5) Suppose that u(t) is the displacement from equilibrium of a mass on a spring, subject to an external force $2\cos(\omega t)$. Thus u satisfies the equation

$$mu'' + \gamma u' + ku = 2\cos(\omega t)$$

a) If $k - m\omega^2 + i\omega\gamma = 2e^{i\pi/4}$, what is $u(t)$ after a long time has passed?
b) Suppose that $k = m = 1$ and the amplitude $A(\omega)$ of the steady state solution has the graph below. What is the approximate value of γ?
c) Suppose that $k = m = 1$ and the maximum amplitude occurs at $\omega = 0$. What can you say about γ?



6) Let $f(x) = x$ for $0 \le x \le 2$. Expand f in a cosine series. Sketch the graph of the function to which the cosine series converges.

7) Solve the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ for $0 \le x \le 1$ with the following initial and boundary conditions:

$$u(x, 0) = 1 \qquad u_x(0, t) = 1 \qquad u_x(1, t) = 2$$

(*Hint:* To satisfy the boundary conditions, look for a particular solution of the differential equation having the form $u(x, t) = t + v(x)$.)