## Resumé of Fourier Series Expansions

1) If $f(x)$ is periodic of period $2\ell$ then

$$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{ik\pi x/\ell} \qquad \text{where} \quad c_k = \frac{1}{2\ell}\int_{-\ell}^{\ell} f(x) e^{-ik\pi x/\ell} dx$$

$$= \frac{a_0}{2} + \sum_{k=1}^{\infty} \left[a_k \cos\left(\frac{k\pi x}{\ell}\right) + b_k \sin\left(\frac{k\pi x}{\ell}\right)\right] \qquad \text{where} \quad a_k = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\cos\left(\frac{k\pi x}{\ell}\right) dx$$

$$b_k = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\sin\left(\frac{k\pi x}{\ell}\right) dx$$

Here and in the rest of this resumé "=" means equal at points of continuity and the middle of the jump at points of discontinuity.

2) If $f(x)$ is odd and periodic of period $2\ell$ then

$$f(x) = \sum_{k=1}^{\infty} b_k \sin\left(\frac{k\pi x}{\ell}\right) \qquad \text{where} \qquad b_k = \frac{2}{\ell}\int_0^{\ell} f(x)\sin\left(\frac{k\pi x}{\ell}\right) dx$$

3) If $f(x)$ is even and periodic of period $2\ell$ then

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos\left(\frac{k\pi x}{\ell}\right) \qquad \text{where} \qquad a_k = \frac{2}{\ell}\int_0^{\ell} f(x)\cos\left(\frac{k\pi x}{\ell}\right) dx$$

4) If $f(x)$ is only defined for $0 < x < \ell$ then

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos\left(\frac{k\pi x}{\ell}\right) \qquad \text{where} \quad a_k = \frac{2}{\ell}\int_0^{\ell} f(x)\cos\left(\frac{k\pi x}{\ell}\right) dx$$

$$= \sum_{k=1}^{\infty} b_k \sin\left(\frac{k\pi x}{\ell}\right) \qquad \text{where} \quad b_k = \frac{2}{\ell}\int_0^{\ell} f(x)\sin\left(\frac{k\pi x}{\ell}\right) dx$$