Basic Trig Identities

(1) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad\csc\theta = \frac{1}{\sin\theta}$ $\quad\sec\theta = \frac{1}{\cos\theta}$ $\quad\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

(2) $\sin(-\theta) = -\sin\theta$ $\quad\cos(-\theta) = \cos\theta$

(3) $\sin(\theta + 2\pi) = \sin\theta$ $\quad\cos(\theta + 2\pi) = \cos\theta$

$\sin(\theta + \pi) = -\sin\theta$ $\quad\cos(\theta + \pi) = -\cos\theta$

$\sin(\frac{\pi}{2} - \theta) = \cos\theta$ $\quad\cos(\frac{\pi}{2} - \theta) = \sin\theta$

(4) $\sin^2\theta + \cos^2\theta = 1$

(5) $\sin(2\theta) = 2\sin\theta\cos\theta$

(6) $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

(7) $\sin(\theta + \varphi) = \sin\theta\cos\varphi + \cos\theta\sin\varphi$

$\cos(\theta + \varphi) = \cos\theta\cos\varphi - \sin\theta\sin\varphi$

More Trig Identities

(4’) $\tan^2\theta + 1 = \sec^2\theta$ $\quad 1 + \cot^2\theta = \csc^2\theta$

(5’,6’) $\tan(2\theta) = \frac{2\tan\theta}{1-2\tan^2\theta}$

(6’) $\cos(2\theta) = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$

$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$

$\tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$

(7’) $\sin(\theta - \varphi) = \sin\theta\cos\varphi - \cos\theta\sin\varphi$

$\cos(\theta - \varphi) = \cos\theta\cos\varphi + \sin\theta\sin\varphi$

$\tan(\theta + \varphi) = \frac{\tan\theta+\tan\varphi}{1-\tan\theta\tan\varphi}$

$\tan(\theta - \varphi) = \frac{\tan\theta-\tan\varphi}{1+\tan\theta\tan\varphi}$

(7”) $\sin\theta\cos\varphi = \frac{1}{2}\{\sin(\theta+\varphi) + \sin(\theta-\varphi)\}$

$\sin\theta\sin\varphi = \frac{1}{2}\{\cos(\theta-\varphi) - \cos(\theta+\varphi)\}$

$\cos\theta\cos\varphi = \frac{1}{2}\{\cos(\theta+\varphi) + \cos(\theta-\varphi)\}$

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

The code here is that, for example, the identities in (3’) are easily derived from the identity in (3). The identity in (5’,6’) is easily derived by dividing the identities in (5) and (6).