Two and Three Dimensional Determinants

The determinant of a 2×2 matrix can be defined by

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$

The right hand side is the product of the diagonal matrix entries minus the product of the off–diagonal matrix entries.

The determinant of a 3×3 matrix can be defined in terms of some 2×2 determinants by

$$\begin{aligned} \det \begin{bmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} &= i \det \begin{bmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} - j \det \begin{bmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} + k \det \begin{bmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} \\ &= i\,(a_2 b_3 - a_3 b_2) \qquad - j\,(a_1 b_3 - a_3 b_1) \qquad + k\,(a_1 b_2 - a_2 b_1) \end{aligned}$$

This formula is called "expansion along the top row". There is one term in the formula for each entry in the top row. The term is a sign times the entry itself times the determinant of the 2×2 matrix gotten by deleting the row and column that contains the entry. The sign alternates, starting with a $+$.