

Math 200 Problem Set IX

- 1) Use polar coordinates to evaluate each of the following integrals.
 - a) $\iint_S (x + y) dx dy$ where S is the region in the first quadrant lying inside the disc $x^2 + y^2 \leq a^2$ and under the line $y = \sqrt{3}x$.
 - b) $\iint_S x dx dy$, where S is the disc segment $x^2 + y^2 \leq 2$, $x \geq 1$.
 - c) $\iint_T (x^2 + y^2) dx dy$ where T is the triangle with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$.
 - d) $\iint_{x^2+y^2 \leq 1} \ln(x^2 + y^2) dx dy$
- 2) Find the volume lying inside the sphere $x^2 + y^2 + z^2 = 2$ and above the paraboloid $z = x^2 + y^2$.
- 3) Find the volume lying inside the cylinder $x^2 + (y - a)^2 = a^2$ and between the upper and lower halves of the cone $z^2 = x^2 + y^2$.
- 4) Find the volume of the region in the first octant below the paraboloid

$$z = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

Hint: use the change of variables $x = au$ and $y = bv$.

- 5) Find the volume common to the cylinders $x^2 + y^2 \leq 2ax$ and $z^2 \leq 2ax$. Use polar coordinates.
- 6) A symmetrical coffee percolator holds 24 cups when full. The interior has a circular cross-section which tapers from a radius of 3" at the centre to 2" at the base and top, which are 12" apart. The bounding surface is parabolic. Where should the mark indicating the 6 cup level be placed?

