Math 200 Problem Set XI

1) Evaluate the volume of a circular cylinder of radius a and height h by means of an integral in spherical coordinates.

2) Use spherical coordinates to find

a) The volume inside the cone $z = \sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 = a^2$.

b) $\iiint_R x\, dV$ and $\iiint_R z\, dV$ over the part of the sphere of radius a that lies in the first octant.

c) The mass of a spherical planet of radius a whose density at distance R from the center is $\rho = A/(B + R^2)$.

d) The volume enclosed by $\rho = a(1 - \cos\phi)$. Here ρ and ϕ refer to the usual spherical coordinates.

3) Find the surface area of

a) The part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the xy plane.

b) The part of the sphere $x^2 + y^2 + z^2 = 4z$ that lies inside the paraboloid $z = x^2 + y^2$.

c) The part of the hyperbolic paraboloid $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

4) A torus of mass M is generated by rotating a circle of radius a about an axis. The axis is in the plane of the circle plane and is a distance b from the centre ($b > a$) of the circle. Find the moment of inertia of the torus about the axis of rotation. By definition the moment of intertia is $\iiint r^2 \rho\, dxdydz$ where ρ is the density at a point in the solid and r is the distance of the point from the axis of rotation. Assume that the density is a constant.