Math 200 Problem Set I

1) Find the equation of a sphere if one of its diameters has end points $(2, 1, 4)$ and $(4, 3, 10)$.

2) Determine whether the given points are collinear (that is, lie on a common straight line).

   a) $(1, 2, 3),\ (0, 3, 7),\ (3, 5, 11)$ b) $(0, 3, -5),\ (1, 2, -3),\ (3, 0, 1)$

3) Show that the set of all points P that are twice as far from $(-1, 5, 3)$ as from $(6, 2, -2)$ is a sphere. Find its centre and radius.

4) Describe and sketch the set of all points in $\mathbb{R}^3$ that satisfy

   a) $z = x$ b) $x^2 + y^2 + z^2 = 2z$ c) $x^2 + z^2 = 4$

   d) $z \geq \sqrt{x^2 + y^2}$ e) $x^2 + y^2 + z^2 = 4,\ z = 1$ f) $x + y + z = 1$

5) The pressure $p(x, y)$ at the point (x, y) is determined by $x^2 - 2px + y^2 + 1 = 0$. Sketch the isobars (level curves of p).

6) Compute the dot product of the vectors $\vec{a}$ and $\vec{b}$. Find the angle between them.

   a) $\vec{a} = (1, 2),\ \vec{b} = (-2, 3)$ b) $\vec{a} = (-1, 1),\ \vec{b} = (1, 1)$

   c) $\vec{a} = (1, 1),\ \vec{b} = (2, 2)$ d) $\vec{a} = (1, 2, 1),\ \vec{b} = (-1, 1, 1)$

   e) $\vec{a} = (-1, 2, 3),\ \vec{b} = (3, 0, 1)$

7) Does the triangle with vertices $(1, 2, 3),\ (4, 0, 5)$ and $(3, 6, 4)$ have a right angle?

8) Use vectors to prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length.

9) Compute $(1, 2, 3) \times (4, 5, 6)$.

10) Prove that

   a) $\hat{\boldsymbol{\imath}} \times \hat{\boldsymbol{\jmath}} = \hat{\mathbf{k}},\ \hat{\boldsymbol{\jmath}} \times \hat{\mathbf{k}} = \hat{\boldsymbol{\imath}},\ \hat{\mathbf{k}} \times \hat{\boldsymbol{\imath}} = \hat{\boldsymbol{\jmath}}$

   b) $\vec{a} \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot (\vec{a} \times \vec{b}) = 0$

   c) $\|\vec{a} \times \vec{b}\|^2 = \|\vec{a}\|^2 \|\vec{b}\|^2 - (\vec{a} \cdot \vec{b})^2$

11 a) Show that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$.

   b) Show that $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c}$.

   c) Derive a formula for $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$ that involves dot but not cross products.