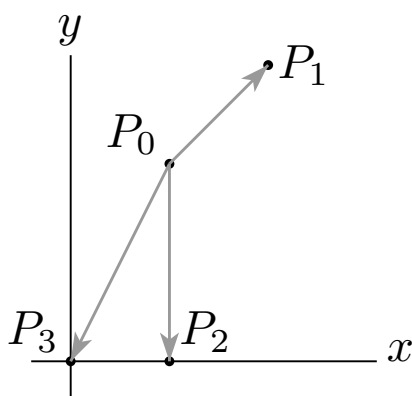


Directional Derivative Example

Question

The directional derivative of some function f at $P_0(1, 2)$ in the direction toward $P_1(2, 3)$ is $2\sqrt{2}$ and in the direction toward $P_2(1, 0)$ is -3 . What is the directional derivative in the direction toward the origin?



Solution

Let $\nabla f(1, 2) = a\hat{i} + b\hat{j}$. Then the directional derivative of f at $P_0(1, 2)$ in the direction \mathbf{v} is

$$D_{\mathbf{v}}f(1, 2) = (a, b) \cdot \frac{\mathbf{v}}{|\mathbf{v}|}$$

In particular, we are told that

$$D_{\overrightarrow{P_0P_1}}f(1, 2) = D_{(1,1)}f(1, 2) = (a, b) \cdot \frac{(1,1)}{\sqrt{2}} = \frac{a}{\sqrt{2}} + \frac{b}{\sqrt{2}} = 2\sqrt{2}$$

$$D_{\overrightarrow{P_0P_2}}f(1, 2) = D_{(0,-2)}f(1, 2) = (a, b) \cdot \frac{(0,-2)}{2} = -b = -3$$

So $b = 3$ and $a + b = 4$ which implies that $\nabla f(1, 2) = (a, b) = (1, 3)$ and, calling the origin P_3 ,

$$D_{\overrightarrow{P_0P_3}}f(1, 2) = D_{(-1,-2)}f(1, 2) = (1, 3) \cdot \frac{(-1,-2)}{\sqrt{5}} = \boxed{-\frac{7}{\sqrt{5}}}$$