The Chain Rule PVT Example

Example 4: Suppose that $F(P,V,T)=0$. Find $\frac{\partial P}{\partial T}$**.**

The variables P, V, T are not independent. They are related by $F(P,V,T)=0$. Given values for any two of P, V, T, the third can be found by solving $F(P,V,T)=0$. We are being asked to find $\frac{\partial P}{\partial T}$. A careful wording of this problem is "The function $P(V,T)$ is defined by $F\big(P(V,T),V,T\big)=0$. Find $\big(\frac{\partial P}{\partial T}\big)_V$".

We are not told explicitly what F is, so we cannot solve explicitly for $P(V,T)$. Instead we differentiate

$$F\big(P(V,T),V,T\big)=0$$

with respect to T, while holding V fixed.

$$\frac{\partial F}{\partial P}\frac{\partial P}{\partial T}+\frac{\partial F}{\partial V}\frac{\partial V}{\partial T}+\frac{\partial F}{\partial T}\frac{\partial T}{\partial T}=0$$

Recalling that V and T are the independent variables and that, in computing $\frac{\partial}{\partial T}$, V is to be held constant,

$$\frac{\partial V}{\partial T}=0 \qquad\qquad \frac{\partial T}{\partial T}=1$$

Now putting in the functional dependence

$$\frac{\partial F}{\partial P}\big(P(V,T),V,T\big)\frac{\partial P}{\partial T}(V,T)+\frac{\partial F}{\partial T}\big(P(V,T),V,T\big)=0$$

and solving

$$\frac{\partial P}{\partial T}(V,T) = -\frac{\frac{\partial F}{\partial T}\big(P(V,T),V,T\big)}{\frac{\partial F}{\partial P}\big(P(V,T),V,T\big)}$$