MATHEMATICS 200 December 2015 Final Exam

1. (a) Consider the plane $4x+2y-4z=3$. Find all parallel planes that are distance 2 from the above plane. Your answers should be in the following form: $4x+2y-4z=C$.

   (b) Find the parametric equation for the line of intersection of the planes

$$x+y+z=11 \qquad \text{and} \qquad x-y-z=13.$$

   (c) Find the tangent plane to

$$\frac{27}{\sqrt{x^2+y^2+z^2+3}}=9$$

   at the point $(2,1,1)$.

2. A function $T(x,y,z)$ at $P=(2,1,1)$ is known to have $T(P)=5$, $T_x(P)=1$, $T_y(P)=2$, and $T_z(P)=3$.

   (a) A bee starts flying at P and flies along the unit vector pointing towards the point $Q=(3,2,2)$. What is the rate of change of $T(x,y,z)$ in this direction?

   (b) Use the linear approximation of T at the point P to approximate $T(1.9,1,1.2)$.

   (c) Let $S(x,y,z)=x+z$. A bee starts flying at P; along which unit vector direction should the bee fly so that the rate of change of $T(x,y,z)$ and of $S(x,y,z)$ are both zero in this direction?

3. Let $w(s,t)=u(2s+3t,3s-2t)$ for some twice differentiable function $u=u(x,y)$.

   (a) Find w_{ss} in terms of u_{xx}, u_{xy} , and u_{yy} (you can assume that $u_{xy}=u_{yx}$).

   (b) Suppose $u_{xx}+u_{yy}=0$. For what constant A will $w_{ss}=Aw_{tt}$?

4. Find and classify the critical points of $f(x,y)=3x^2y+y^3-3x^2-3y^2+4$.

5. Use Lagrange multipliers to find the minimum and maximum values of $(x+z)e^y$ subject to $x^2+y^2+z^2=6$.

6. Consider the domain D above the x–axis and below parabola $y=1-x^2$ in the xy–plane.

   (a) Sketch D.

   (b) Express

$$\iint_D f(x,y)\,\mathrm{d}A$$

   as an iterated integral corresponding to the order $\mathrm{d}x\,\mathrm{d}y$. Then express this integral as an iterated integral corresponding to the order $\mathrm{d}y\,\mathrm{d}x$.

   (c) Compute the integral in the case $f(x,y)=e^{x-(x^3/3)}$.

7. Let E be the region inside the cylinder $x^2 + y^2 = 1$, below the plane $z = y$ and above the plane $z = -1$. Express the integral

$$\iiint_E f(x, y, z)\ \mathrm{d}V$$

as three different iterated integrals corresponding to the orders of integration: (a) $\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$, (b) $\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$, and (c) $\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}x$.

8. The solid E is bounded below by the paraboloid $z = x^2 + y^2$ and above by the cone $z = \sqrt{x^2 + y^2}$. Let

$$I = \iiint_E z\left(x^2 + y^2 + z^2\right)\ \mathrm{d}V$$

(a) Write I in terms of cylindrical coordinates. Do not evaluate.

(b) Write I in terms of spherical coordinates. Do not evaluate.

(c) Calculate I.