MATHEMATICS 200 December 2015 Final Exam

1. (a) Consider the plane $4x + 2y - 4z = 3$. Find all parallel planes that are distance 2 from the above plane. Your answers should be in the following form: $4x + 2y - 4z = C$.
(b) Find the parametric equation for the line of intersection of the planes

$$x + y + z = 11 \quad \text{and} \quad x - y - z = 13.$$  

(c) Find the tangent plane to $$\frac{27}{\sqrt{x^2 + y^2 + z^2 + 3}} = 9$$ at the point $(2, 1, 1)$.

2. A function $T(x, y, z)$ at $P = (2, 1, 1)$ is known to have $T(P) = 5$, $T_x(P) = 1$, $T_y(P) = 2$, and $T_z(P) = 3$.
   (a) A bee starts flying at $P$ and flies along the unit vector pointing towards the point $Q = (3, 2, 2)$. What is the rate of change of $T(x, y, z)$ in this direction?
   (b) Use the linear approximation of $T$ at the point $P$ to approximate $T(1.9, 1, 1.2)$.
   (c) Let $S(x, y, z) = x + z$. A bee starts flying at $P$, along which unit vector direction should the bee fly so that the rate of change of $T(x, y, z)$ and of $S(x, y, z)$ are both zero in this direction?

3. Let $w(s, t) = u(2s + 3t, 3s - 2t)$ for some twice differentiable function $u = u(x, y)$.
   (a) Find $w_{ss}$ in terms of $u_{xx}$, $u_{xy}$, and $u_{yy}$ (you can assume that $u_{xy} = u_{yx}$).
   (b) Suppose $u_{xx} + u_{yy} = 0$. For what constant $A$ will $w_{ss} = Aw_{tt}$?

4. Find and classify the critical points of $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 4$.

5. Use Lagrange multipliers to find the minimum and maximum values of $(x + z)e^y$ subject to $x^2 + y^2 + z^2 = 6$.

6. Consider the domain $D$ above the $x$–axis and below parabola $y = 1 - x^2$ in the $xy$–plane.
   (a) Sketch $D$.
   (b) Express $$\int\int_D f(x, y) \, dA$$ as an iterated integral corresponding to the order $dx \, dy$. Then express this integral as an iterated integral corresponding to the order $dy \, dx$.
   (c) Compute the integral in the case $f(x, y) = e^{x-(x^2/3)}$. 

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7. Let \( E \) be the region inside the cylinder \( x^2 + y^2 = 1 \), below the plane \( z = y \) and above the plane \( z = -1 \). Express the integral

\[
\iiint_E f(x, y, z) \, dV
\]

as three different iterated integrals corresponding to the orders of integration: (a) \( dz \, dx \, dy \), (b) \( dx \, dy \, dz \), and (c) \( dy \, dz \, dx \).

8. The solid \( E \) is bounded below by the paraboloid \( z = x^2 + y^2 \) and above by the cone \( z = \sqrt{x^2 + y^2} \). Let

\[
I = \iiint_E z(x^2 + y^2 + z^2) \, dV
\]

(a) Write \( I \) in terms of cylindrical coordinates. Do not evaluate.
(b) Write \( I \) in terms of spherical coordinates. Do not evaluate.
(c) Calculate \( I \).