1. Short Answer Problems. Show your work. Not all questions are of equal difficulty. Simplify your answers as much as possible in this question.

(a) The line $L$ has vector parametric equation $r(t) = (2 + 3t)i + 4tj - k$.
   
   i. Write the symmetric equations for $L$.
   
   ii. Let $\alpha$ be the angle between the line $L$ and the plane given by the equation $x - y + 2z = 0$. Find $\alpha$.

(b) i. Find the equation of the tangent plane to the surface $x^2z^3 + y\sin(\pi x) = -y^2$ at the point $P = (1, 1, -1)$.
   
   ii. Let $z$ be defined implicitly by $x^2z^3 + y\sin(\pi x) = -y^2$. Find $\frac{\partial z}{\partial x}$ at the point $P = (1, 1, -1)$.
   
   iii. Let $z$ be the same implicit function as in part (ii), defined by the equation $x^2z^3 + y\sin(\pi x) = -y^2$. Let $x = 0.97$, and $y = 1$. Find the approximate value of $z$.

(c) Suppose that $u = x^2 + yz$, $x = \rho r \cos(\theta)$, $y = \rho r \sin(\theta)$ and $z = \rho r$. Find $\frac{\partial u}{\partial \rho}$ at the point $(\rho_0, r_0, \theta_0) = (2, 3, \pi/2)$.

(d) Let $f(x)$ be a differentiable function, and suppose it is given that $f'(0) = 10$. Let $g(s, t) = f(as - bt)$, where $a$ and $b$ are constants. Evaluate $\frac{\partial g}{\partial s}$ at the point $(s, t) = (b, a)$, that is, find $\frac{\partial g}{\partial s}|_{(b,a)}$.

(e) Suppose it is known that the direction of the fastest increase of the function $f(x, y)$ at the origin is given by the vector $\langle 1, 2 \rangle$. Find a unit vector $u$ that is tangent to the level curve of $f(x, y)$ that passes through the origin.

(f) Find all the points on the surface $x^2 + 9y^2 + 4z^2 = 17$ where the tangent plane is parallel to the plane $x - 8z = 0$.

(g) Find the total mass of the rectangular box $[0, 1] \times [0, 2] \times [0, 3]$ (that is, the box defined by the inequalities $0 \leq x \leq 1, 0 \leq y \leq 2, 0 \leq z \leq 3$), with density function $h(x, y, z) = x$.

2. The shape of a hill is given by $z = 1000 - 0.02x^2 - 0.01y^2$. Assume that the $x$–axis is pointing East, and the $y$–axis is pointing North, and all distances are in metres.

(a) What is the direction of the steepest ascent at the point $(0, 100, 900)$? (The answer should be in terms of directions of the compass).

(b) What is the slope of the hill at the point $(0, 100, 900)$ in the direction from (a)?

(c) If you ride a bicycle on this hill in the direction of the steepest descent at 5 m/s, what is the rate of change of your altitude (with respect to time) as you pass through the point $(0, 100, 900)$?
3. (a) Find the minimum of the function

\[ f(x, y, z) = (x - 2)^2 + (y - 1)^2 + z^2 \]

subject to the constraint \( x^2 + y^2 + z^2 = 1 \), using the method of Lagrange multipliers.

(b) Give a geometric interpretation of this problem.

4. (a) Find the minimum of the function \( h(x, y) = -4x - 2y + 6 \) on the closed bounded domain defined by \( x^2 + y^2 \leq 1 \).

(b) Explain why Question 4 gives another way of solving Question 3.

5. This question is about the integral

\[ \int_0^1 \int_{\sqrt{3y}}^{\sqrt{4-y^2}} \ln(1 + x^2 + y^2) \, dx \, dy \]

(a) Sketch the domain of integration.

(b) Evaluate the integral by transforming to polar coordinates.

6. Evaluate

\[ \int_{-1}^{0} \int_{-2}^{2x} e^{y^2} \, dy \, dx \]

7. Let \( a > 0 \) be a fixed positive real number. Consider the solid inside both the cylinder \( x^2 + y^2 = ax \) and the sphere \( x^2 + y^2 + z^2 = a^2 \). Compute its volume.

Hint: \( \int \sin^3(\theta) = \frac{1}{12} \cos(3\theta) - \frac{3}{4} \cos(\theta) + C \)

8. (a) Sketch the surface given by the equation \( z = 1 - x^2 \).

(b) Let \( E \) be the solid bounded by the plane \( y = 0 \), the cylinder \( z = 1 - x^2 \), and the plane \( y = z \). Set up the integral

\[ \iiint_E f(x, y, z) \, dV \]

as an iterated integral.

9. (a) Find the volume of the solid inside the surface defined by the equation \( \rho = 8 \sin(\varphi) \) in spherical coordinates.

Hint: you do not need a sketch to answer this question; and

\[ \int \sin^4(\varphi) = \frac{1}{32} (12\varphi - 8 \sin(2\varphi) + \sin(4\varphi)) + C \]

(b) Sketch this solid or describe what it looks like.

Hint: it is a solid of revolution.