

MATHEMATICS 200 April 2007 Final Exam

- A plane Π passes through the points $A = (1, 1, 3)$, $B = (2, 0, 2)$ and $C = (2, 1, 0)$ in \mathbb{R}^3 .
 - Find an equation for the plane Π .
 - Find the point E in the plane Π such that the line L through $D = (6, 1, 2)$ and E is perpendicular to Π .
- Consider the function f that maps each point (x, y) in \mathbb{R}^2 to ye^{-x} .
 - Suppose that $x = 1$ and $y = e$, but errors of size 0.1 are made in measuring each of x and y . Estimate the maximum error that this could cause in $f(x, y)$.
 - The graph of the function f sits in \mathbb{R}^3 , and the point $(1, e, 1)$ lies on that graph. Find a nonzero vector that is perpendicular to that graph at that point.
- A mosquito is at the location $(3, 2, 1)$ in \mathbb{R}^3 . She knows that the temperature T near there is given by $T = 2x^2 + y^2 - z^2$.
 - She wishes to stay at the same temperature, but must fly in some initial direction. Find a direction in which the initial rate of change of the temperature is 0.
 - If you and another student both get correct answers in part (a), must the directions you give be the same? Why or why not?
 - What initial direction or directions would suit the mosquito if she wanted to cool down as fast as possible?
- Let F be a function on \mathbb{R}^2 . Denote points in \mathbb{R}^2 by (u, v) and the corresponding partial derivatives of F by $F_u(u, v)$, $F_v(u, v)$, $F_{uu}(u, v)$, $F_{uv}(u, v)$, etc.. Assume those derivatives are all continuous. Express

$$\frac{\partial^2}{\partial x \partial y} F(x^2 - y^2, 2xy)$$

in terms of partial derivatives of the function F .

Hint: Let $u = x^2 - y^2$, and $v = 2xy$.

- Find all critical points for $f(x, y) = x(x^2 + xy + y^2 - 9)$. Also find out which of these points give local maximum values for $f(x, y)$, which give local minimum values, and which give saddle points.
- Find the largest and smallest values of x^2y^2z in the part of the plane $2x + y + z = 5$ where $x \geq 0$, $y \geq 0$ and $z \geq 0$. Also find all points where those extreme values occur.
- A region E in the xy -plane has the property that for all continuous functions f

$$\iint_E f(x, y) \, dA = \int_{x=-1}^{x=3} \left[\int_{y=x^2}^{y=2x+3} f(x, y) \, dy \right] dx$$

- (a) Compute $\iint_E x \, dA$.
- (b) Sketch the region E .
- (c) Set up $\iint_E x \, dA$ as an integral or sum of integrals in the opposite order.
8. A certain solid V is a right-circular cylinder. Its base is the disk of radius 2 centred at the origin in the xy -plane. It has height 2 and density $\sqrt{x^2 + y^2}$.
- A smaller solid U is obtained by removing the inverted cone, whose base is the top surface of V and whose vertex is the point $(0, 0, 0)$.
- (a) Use cylindrical coordinates to set up an integral giving the mass of U .
- (b) Use spherical coordinates to set up an integral giving the mass of U .
- (c) Find that mass.