1. Consider the surface given by:

\[ z^3 - xyz^2 - 4x = 0. \]

(a) Find expressions for \( \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} \) as functions of \( x, y, z \).

(b) Evaluate \( \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} \) at \( (1, 1, 2) \).

(c) Measurements are made with errors, so that \( x = 1 \pm 0.03 \) and \( y = 1 \pm 0.02 \). Find the corresponding maximum error in measuring \( z \).

(d) A particle moves over the surface along the path whose projection in the \( xy \)-plane is given in terms of the angle \( \theta \) as

\[ x(\theta) = 1 + \cos \theta, \ y(\theta) = \sin \theta \]

from the point \( A : x = 2, \ y = 0 \) to the point \( B : x = 1, \ y = 1 \). Find \( \frac{dz}{d\theta} \) at points \( A \) and \( B \).

2. A hiker is walking on a mountain with height above the \( z = 0 \) plane given by

\[ z = f(x, y) = 6 - xy^2 \]

The positive \( x \)-axis points east and the positive \( y \)-axis points north, and the hiker starts from the point \( P(2, 1, 4) \).

(a) In what direction should the hiker proceed from \( P \) to ascend along the steepest path? What is the slope of the path?

(b) Walking north from \( P \), will the hiker start to ascend or descend? What is the slope?

(c) In what direction should the hiker walk from \( P \) to remain at the same height?

3. (a) Find and classify all critical points of the function

\[ f(x, y) = x^3 - y^3 - 2xy + 6. \]

(b) Use the method of Lagrange Multipliers to find the maximum and minimum values of

\[ f(x, y) = xy \]

subject to the constraint

\[ x^2 + 2y^2 = 1. \]

4. The integral \( I \) is defined as

\[ I = \int \int_R f(x, y) \, dA = \int_1^{\sqrt{2}} \int_{1/y}^{\sqrt{2}} f(x, y) \, dx \, dy + \int_1^{\sqrt{2}} \int_{1/y}^{\sqrt{2}} f(x, y) \, dx \, dy \]
(a) Sketch the region \( R \).
(b) Re–write the integral \( I \) by reversing the order of integration.
(c) Compute the integral \( I \) when \( f(x, y) = x/y \).

5. (a) Sketch the region \( \mathcal{L} \) (in the first quadrant of the \( xy \)-plane) with boundary curves

\[
x^2 + y^2 = 2, \quad x^2 + y^2 = 4, \quad y = x, \quad y = 0.
\]

The mass of a thin lamina with a density function \( \rho(x, y) \) over the region \( \mathcal{L} \) is given by

\[
M = \iint_{\mathcal{L}} \rho(x, y) \, dA
\]

(b) Find an expression for \( M \) as an integral in polar coordinates.
(c) Find \( M \) when

\[
\rho(x, y) = \frac{2xy}{x^2 + y^2}
\]

6. (a) A triple integral \( \iiint_{E} f(x, y, z) \, dV \) is given in the iterated form

\[
J = \int_{0}^{1} \int_{0}^{1-x^2} \int_{0}^{4x-4} f(x, y, z) \, dy \, dz \, dx
\]

(i) Sketch the domain \( E \) in 3–dimensions. Be sure to show the units.
(ii) Rewrite the integral as one or more iterated integrals in the form

\[
J = \int_{y=}^{y=} \int_{x=}^{x=} \int_{z=}^{z=} f(x, y, z) \, dz \, dx \, dy
\]

(b) Use spherical coordinates to evaluate the integral

\[
I = \iiint_{D} z \, dV
\]

where \( D \) is the solid enclosed by the cone \( z = \sqrt{x^2 + y^2} \) and the sphere \( x^2 + y^2 + z^2 = 4 \).