MATHEMATICS 200 December 2014 Final Exam Answers

1. $\boldsymbol{\nabla} f(a,b,c) = \sqrt{3}\,\langle 2, 6, -4\rangle$

2. See the solution set.

3. (a) The differential at $x = a$, $y = b$ is $\frac{\mathrm{d}x}{e^{f(a,b)}+b} + \frac{1-f(a,b)}{e^{f(a,b)}+b}\,\mathrm{d}y$

   (b) $f\big(0.99\,,\,0.01\big) \approx 0$

4. $\big(\frac{1}{\sqrt{2}}\,,\,-1\,,\,-\frac{1}{2}\big)$ and $\big(-\frac{1}{\sqrt{2}}\,,\,-1\,,\,-\frac{1}{2}\big)$

5. (a) (0,0) and (3,0) and (0,3) are saddle points
   (1,1) is a local min

   (b) The minimum is -1 at $(1,1)$ and the maximum is 80 at $(4,4)$.

6. (a) $D = \Big\{\ (r\cos\theta\,,\,r\sin\theta)\ \Big|\ -\pi/2 \le \theta \le \pi/4,\ 0 \le r \le 2\cos\theta\ \Big\}$

   (b) Volume $= \frac{40}{18\sqrt{2}} + \frac{16}{9}$

7. (a) The unit vector in the direction of maximum rate of increse is $\frac{1}{\sqrt{10}}(3,0,1)$.

   (b) 2π

8. (a) $\int_0^a \mathrm{d}z \int_\pi^{3\pi/2} \mathrm{d}\theta \int_0^{\sqrt{a^2-z^2}} \mathrm{d}r\ r\big(r^2+z^2\big)^{2014}$ (b) $\int_0^{\pi/2} \mathrm{d}\varphi \int_\pi^{3\pi/2} \mathrm{d}\theta \int_0^a \mathrm{d}\rho\ \rho^{4030}\sin\varphi$

   (c) $\frac{a^{4031}\pi}{8062}$

9. $\int_{y=-\sqrt{6}}^{y=\sqrt{6}} \int_{x=y^2/2}^{x=3} \int_{z=y^2}^{z=2x} f(x,y,z)\ \mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$

   $\int_{y=-\sqrt{6}}^{y=\sqrt{6}} \int_{z=y^2}^{z=6} \int_{x=z/2}^{x=3} f(x,y,z)\ \mathrm{d}x\,\mathrm{d}z\,\mathrm{d}y$

   $\int_{z=0}^{z=6} \int_{x=z/2}^{x=3} \int_{y=-\sqrt{z}}^{y=\sqrt{z}} f(x,y,z)\ \mathrm{d}y\,\mathrm{d}x\,\mathrm{d}z$