MATHEMATICS 200 April 2014 Final Exam Answers

1. (a) $(x, y, z) = (-3, 1, 0) + t\,\langle 0, -1, 1\rangle$ (b) $\sqrt{17}$ (c) $2\sqrt{19}$

2. (a) -16 (b) $C\,\langle 4\,,\,1\,,\,-2\rangle$ for any nonzero constant C

   (c) Any positive non zero multiple of $-\,\langle 1\,,\,2\,,\,1\rangle$ will do.

3. *Case* $k < \frac{1}{2}$*:*

critical point	type
$(0, 0)$	local max
$(0, 2)$	saddle point

   Case $k = \frac{1}{2}$*:*

critical point	type
$(0, 0)$	local max
$(0, 2)$	unknown

   Case $k > \frac{1}{2}$*:*

critical point	type
$(0, 0)$	local max
$(0, 2)$	local min
$\left(\sqrt{\frac{1}{k^3}(2k-1)}\,,\,\frac{1}{k}\right)$	saddle point
$\left(-\sqrt{\frac{1}{k^3}(2k-1)}\,,\,\frac{1}{k}\right)$	saddle point

4. The min is $-12\sqrt{3}$ at $(-3\sqrt{3}, 0, 3)$ and the max is 48 at $(4, \pm 4, 2)$.

5. $\frac{\sqrt{3}}{2} - \frac{\pi}{6}$

6. (a) $I = \int_0^2 \int_y^{6-y^2} f(x, y)\,\mathrm{d}x\,\mathrm{d}y$ (b) $J = \int_0^1 \int_z^1 \int_y^1 f(x, y, z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

7. $\frac{16\pi}{3}$

8. (a) $I = \int_0^1 \mathrm{d}r \int_0^{2\pi} \mathrm{d}\theta \int_0^r \mathrm{d}z\; r\; z\; \sqrt{r^2 + z^2}$ (b) $I = \int_{\pi/4}^{\pi/2} \mathrm{d}\varphi \int_0^{2\pi} \mathrm{d}\theta \int_0^{1/\sin\varphi} \mathrm{d}\rho\; \rho^4 \sin\varphi\; \cos\varphi$

   (c) $\frac{2(2\sqrt{2}-1)\pi}{15}$