

Example III.19 (Feldman's notes)

Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$. We look for a matrix B that obeys $AB = I$. The columns of I are $\hat{\mathbf{e}}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\hat{\mathbf{e}}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Denote the columns of B by \vec{B}_1 and \vec{B}_2 . That is

$$B = \begin{bmatrix} X & X' \\ Y & Y' \end{bmatrix} = [\vec{B}_1 \ \vec{B}_2] \quad \vec{B}_1 = \begin{bmatrix} X \\ Y \end{bmatrix} \quad \vec{B}_2 = \begin{bmatrix} X' \\ Y' \end{bmatrix}$$

We are to find a B obeying

$$AB = \begin{bmatrix} X+Y & X'+Y' \\ X+2Y & X'+2Y' \end{bmatrix} = [A\vec{B}_1 \ A\vec{B}_2] = [\hat{\mathbf{e}}_1 \ \hat{\mathbf{e}}_2]$$

or, equivalently, $A\vec{B}_1 = \hat{\mathbf{e}}_1$, $A\vec{B}_2 = \hat{\mathbf{e}}_2$. We wish to solve for \vec{B}_1 and \vec{B}_2 . The augmented matrices for these two systems of equations are $[A|\hat{\mathbf{e}}_1]$ and $[A|\hat{\mathbf{e}}_2]$. For efficiency, we combine them into a single larger augmented matrix

$$[A|\hat{\mathbf{e}}_1 \ \hat{\mathbf{e}}_2] = \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array} \right]$$

We can apply row operations to bring this to reduced row echelon form

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{array} \right] (2) - (1) \qquad \left[\begin{array}{cc|cc} 1 & 0 & 2 & -1 \\ 0 & 1 & -1 & 1 \end{array} \right] (1) - (2)$$

The augmented matrix on the right represents the two systems of equations $I\vec{B}_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $I\vec{B}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$. Thus

$$\vec{B}_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \quad \vec{B}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad B = [\vec{B}_1 \ \vec{B}_2] = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$