Substitutions for Integrating Trigonometric Functions

y	dy		good for
$\sin x$	$\cos x\,dx$	$\sin x = y,\ \cos^2 x = 1 - y^2$	$\int \sin^m x\,\cos^n x\,dx$ with $n \geq 1,$ odd
$\cos x$	$-\sin x\,dx$	$\sin^2 x = 1 - y^2,\ \cos x = y$	$\int \sin^m x\,\cos^n x\,dx$ with $m \geq 1,$ odd
$\tan x$	$\sec^2 x\,dx$	$\tan x = y,\ \sec^2 x = 1 + y^2$	$\int \tan^m x\,\sec^n x\,dx$ with $n \geq 2,$ even
$\cot x$	$-\csc^2 x\,dx$	$\cot x = y,\ \csc^2 x = 1 + y^2$	$\int \cot^m x\,\csc^n x\,dx$ with $n \geq 2,$ even
$\sec x$	$\sec x\,\tan x\,dx$	$\sec x = y,\ \tan^2 x = y^2 - 1$	$\int \tan^m x\,\sec^n x\,dx$ with $m \geq 1,$ odd, $n \geq 1$
$\csc x$	$-\csc x\,\cot x\,dx$	$\csc x = y,\ \cot^2 x = y^2 - 1$	$\int \cot^m x\,\csc^n x\,dx$ with $m \geq 1,$ odd, $n \geq 1$

Substitutions for Integrating Square Roots

Integral	Substitution		
$\int (a + bx^2)^{m/2}\,x^n\,dx$ with $n \geq 1$, odd	$y = a + bx^2$	$dy = 2bx\,dx$	$a + bx^2 = y,\ x^2 = \frac{1}{b}(y - a)$
$\int (a^2 - x^2)^{m/2}\,x^n\,dx$ with $n \geq 0$, even	$x = a\sin y$	$dx = a\cos y\,dy$	$\sqrt{a^2 - x^2} = \lvert a\cos y\rvert$
$\int (a^2 - b^2x^2)^{m/2}\,x^n\,dx$ with $n \geq 0$, even	$bx = a\sin y$	$dx = \frac{a}{b}\cos y\,dy$	$\sqrt{a^2 - b^2x^2} = \lvert a\cos y\rvert$
$\int (a^2 + x^2)^{m/2}\,x^n\,dx$ with $n \geq 0$, even	$x = a\tan y$	$dx = a\sec^2 y\,dy$	$\sqrt{a^2 + x^2} = \lvert a\sec y\rvert$
$\int (a^2 + b^2x^2)^{m/2}\,x^n\,dx$ with $n \geq 0$, even	$bx = a\tan y$	$dx = \frac{a}{b}\sec^2 y\,dy$	$\sqrt{a^2 + b^2x^2} = \lvert a\sec y\rvert$
$\int (x^2 - a^2)^{m/2}\,x^n\,dx$ with $n \geq 0$, even	$x = a\sec y$	$dx = a\sec y\,\tan y\,dy$	$\sqrt{x^2 - a^2} = \lvert a\tan y\rvert$
$\int (b^2x^2 - a^2)^{m/2}\,x^n\,dx$ with $n \geq 0$, even	$bx = a\sec y$	$dx = \frac{a}{b}\sec y\,\tan y\,dy$	$\sqrt{b^2x^2 - a^2} = \lvert a\tan y\rvert$