

MATHEMATICS 121, Problem Set V

Due Tuesday, March 25

- 1) For each of the following (a) sketch the curve and (b) eliminate the parameter to find the Cartesian equation of the curve.

a) $x = \sin^2 \theta, y = \cos^2 \theta$

b) $x = \cos t, y = \cos 2t$

- 2) Describe the motion of a particle with position $(x, y) = (2 \sin t, 3 \cos t)$ as t varies from 0 to 2π .

- 3) Match the parametric equations with the graphs (labelled I-VI). Give reasons for your choices.

a) $x = t^3 - 2t, y = t^2 - t$

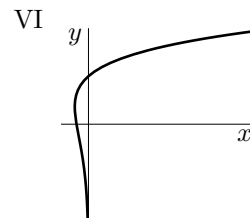
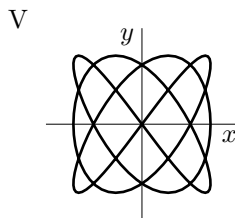
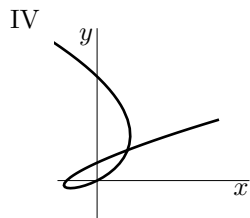
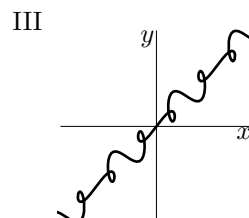
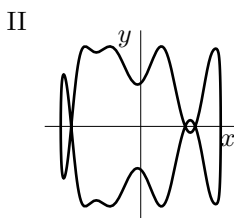
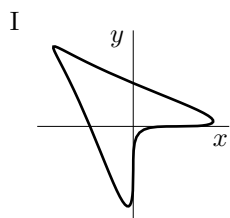
b) $x = t^4 - t^2, y = t + \ln t$

c) $x = \sin 3t, y = \sin 4t$

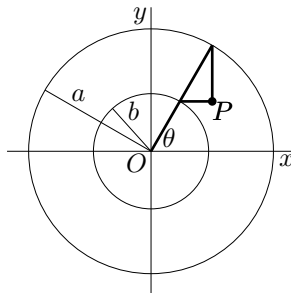
d) $x = t + \sin 2t, y = t + \sin 3t$

e) $x = \sin(t + \sin t), y = \cos(t + \cos t)$

f) $x = \cos t, y = \sin(t + \sin 5t)$



- 4) Find the parametric equations for the set of all points P determined as shown in the figure, using the angle θ as the parameter. Then eliminate the parameter and identify the curve.



- 5) Find an equation of the tangent line to the specified curve at the specified point by two methods: (i) without eliminating the parameter and (ii) by first eliminating the parameter.

a) $x = 2t + 3, y = t^2 + 2t, (5, 3)$

b) $x = 5 \cos t, y = 5 \sin t, (3, 4)$

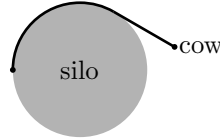
- 6) Find the points on the curve $x = t^3 - 3t^2, y = t^3 - 3t$ where the tangent is horizontal or vertical. Using an analysis of the intervals in which the curve rises and falls, sketch the curve.

- 7) At what point does the curve $x = 1 - 2 \cos^2 t, y = (\tan t)(1 - 2 \cos^2 t)$ cross itself. Find the equations of both tangents at that point.

- 8) Find the equations of the tangents to the curve $x = 3t^2 + 1, y = 2t^3 + 1$ that pass through the point $(4, 3)$.

- 9) Use the parametric equations of an ellipse, $x = a \cos \theta, y = b \sin \theta$, to find the area that it encloses.

- 10) A string is wound around a circle of radius r and then unwound while being held taut. Find the equation of the curve traced by the point P at the end of the string if the initial position of P is $(r, 0)$.
- 11) A cow is tied to a silo with radius r by a rope that is just long enough to reach the opposite side of the silo. Find the area available for grazing by the cow.



- 12) At 2:00 a.m. the animal, *studentia enubriatus*, having contracted the disease, *munchia*, leaves its dwelling to forage for food. At time t , its x -coordinate (in m) is $60e^t \sin 2\pi t$ and its y -coordinate (in m) is $60e^t \cos 2\pi t$. (The time t is in hours and $t = 0$ corresponds to 2:00 a.m.) If it still has not found any food at 4:00 a.m.
- sketch the path that it has followed and
 - determine how far has it travelled.
- 13) Find the length of the parametric curve $x = \frac{3}{4}t^4$, $y(t) = \frac{1}{6}t^6$, $t \geq 0$ from the point $(0, 0)$ to the point $(12, \frac{32}{3})$.
- 14) Find the total length of the astroid $x = a \cos^3 \theta$, $y = a \sin^3 \theta$.
- 15) Set up, but do not evaluate, an integral representing the area of the surface obtained by rotating the given curve about the x -axis.
- $x = t^3$, $y = t^4$, $0 \leq t \leq 1$
 - $x = a \cos^3 \theta$, $y = a \sin^3 \theta$, $0 \leq \theta \leq \frac{\pi}{2}$

Reminder: Quiz V is on Tuesday, April 1. It will cover up to the end of the class of Friday, March 28.

Reminder: The final exam is on Tuesday, April 15 at 8:30 a.m.