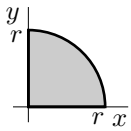
MATHEMATICS 121, Problem Set III
Due Tuesday, February 11

1) Masses $m_1 = 2$, $m_2 = 3$ and $m_3 = 1$ are located at $(5, 1)$, $(3, -2)$ and $(-2, 4)$ respectively. Find the centre of mass of the system of three masses.

2) Find the centroid of the region bounded by the curves $y = 3x + 5$, $y = 0$, $x = -1$ and $x = 2$.

3) Find the centroid of the region bounded by the curves $y = \sin x$, $y = 0$, $x = 0$ and $x = \frac{\pi}{2}$.

4) Find the centre of mass of the membrane with density $\rho = 2$ and shape 

5) Use the Theorem of Pappus to find the volume of a cone with height h and base radius r.

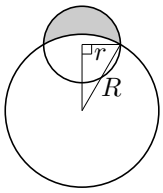
6) Evaluate the given integral.

a) $\int_{1/2}^{\sqrt{3}/2} \frac{1}{x^2\sqrt{1-x^2}}\,dx$ b) $\int_0^2 x^3\sqrt{4-x^2}\,dx$ c) $\int_0^2 \frac{x^3}{\sqrt{x^2+4}}\,dx$

d) $\int_0^3 \frac{1}{\sqrt{9+x^2}}\,dx$ e) $\int_0^3 x^2\sqrt{9-x^2}\,dx$ f) $\int \frac{dx}{\sqrt{x^2+4x+8}}$

g) $\int \frac{dx}{(5-4x-x^2)^{5/2}}$ h) $\int e^t\sqrt{9-e^{2t}}\,dt$

7) Find the area of the crescent shaped region (called "a lune") bounded by arcs of circles with radii r and R.

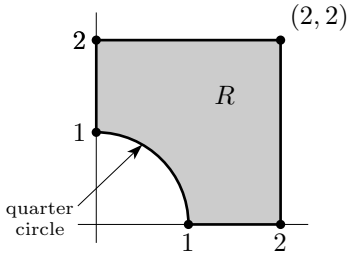


8) Let R be the region in the xy–plane bounded by $y = x\sin x,\ y = 0,\ x = 0$ and $x = \pi$.
   a) Find the volume of the solid obtained by rotating R about the y–axis.
   b) Find the volume of the solid obtained by rotating R about the x–axis.

9) Let R be the region bounded by the line $y = 2$ and $y = (x-4)^2 - 2$. Calculate the position of the centroid of R.

10) Find the x–coordinate of the centroid of the plane region R that lies in the first quadrant $x \geq 0,\ y \geq 0$ and inside the ellipse $4x^2 + 9y^2 = 36$. (The area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab square units.)

11) Find the centroid of the region R in the diagram.



Reminder: The Midterm is on Tuesday, February 25. It will cover up to the end of the class of Tuesday, February 11.