

MATHEMATICS 121, Problem Set II

Due Tuesday, January 28

1) Evaluate the given indefinite integral by making the given substitution.

a) $\int \frac{x^2}{\sqrt{2+x^3}} dx, u = 2 + x^3$

b) $\int \frac{dx}{(2x+1)^2}, u = 2x + 1$

2) Evaluate the given integral.

a) $\int x^3(1-x^4)^5 dx$

b) $\int \frac{dt}{(1-3t)^4}$

c) $\int \sec^2 3\theta d\theta$

d) $\int x^5 \sqrt[3]{x^3+1} dx$

e) $\int \frac{e^x}{e^x+1} dx$

f) $\int_0^{\frac{\pi}{4}} \sin 4t dt$

g) $\int_0^{\frac{\pi}{3}} \frac{\sin \theta}{\cos^2 \theta} d\theta$

h) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \sin x}{1+x^6} dx$

i) $\int_0^{\frac{1}{2}} \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$

j) $\int \frac{dx}{e^x+e^{-x}}$

k) $\int_0^{\pi/2} \sqrt{1+\cos x} dx$

3) Use the method of cylindrical shells to find the volume generated by rotating the region bounded by $y = e^{-x^2}$, $y = 0$, $x = 0$ and $x = 1$ about the y -axis.

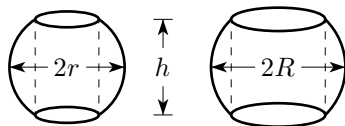
4) Use the method of cylindrical shells to find the volume generated by rotating the region bounded by $y^2 - 6y + x = 0$, and $x = 0$ about the x -axis.

5) Use the method of cylindrical shells to find the volume generated by rotating the region bounded by $y = 4x - x^2$ and $y = 8x - 2x^2$ about $x = -2$.

6) Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by $x = 4 - y^2$ and $x = 8 - 2y^2$ about $y = 5$.

7) Describe a solid whose volume is $\int_0^{\pi} 2\pi(4-x)\sin^4 x dx$.

8) Suppose you make two napkin rings by drilling holes with different diameters through two wooden balls — one of radius r and the other of radius R , with $R > r$. The hole diameters are chosen so that both napkin rings have height h . Which ring has more wood in it?



9) The region below the curve $y = \frac{1}{\sqrt{1-x^2}}$, above the x -axis and between the y -axis and the line $x = \frac{1}{2}$ is rotated about the line $x = 2$. Find the volume of the solid of revolution so obtained.

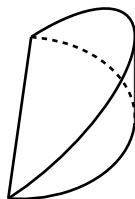
10) Let \mathcal{R} be the plane region bounded by $x = 0$, $x = 1$, $y = 0$ and $y = c\sqrt{1+x^2}$, where c is a positive constant.

(a) Find the volume V_1 of the solid obtained by revolving \mathcal{R} about the x -axis.

(b) Find the volume V_2 of the solid obtained by revolving \mathcal{R} about the y -axis.

(c) If $V_1 = V_2$, what is the value of c ?

11) A 45° notch is cut to the centre of a cylindrical log having radius 20 cm. One plane face of the notch is perpendicular to the axis of the log. What volume of wood was removed?



Reminder: Quiz II is on Tuesday, February 4. It will cover up to the end of the class of Friday, January 31.