Table of Derivatives

Throughout this table, a and b are constants, independent of x.

$F(x)$	$F'(x) = \frac{dF}{dx}$
$af(x) + bg(x)$	$af'(x) + bg'(x)$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x) - g(x)$	$f'(x) - g'(x)$
$af(x)$	$af'(x)$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$f(x)g(x)h(x)$	$f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x)-f(x)g'(x)}{g(x)^2}$
$\frac{1}{g(x)}$	$-\frac{g'(x)}{g(x)^2}$
$f\big(g(x)\big)$	$f'\big(g(x)\big)g'(x)$
a	0
x	1
x^a	ax^{a-1}
$g(x)^a$	$ag(x)^{a-1}g'(x)$
$\sin x$	$\cos x$
$\sin g(x)$	$g'(x) \cos g(x)$
$\cos x$	$-\sin x$
$\cos g(x)$	$-g'(x) \sin g(x)$
$\tan x$	$\sec^2 x$
$\csc x$	$-\csc x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\csc^2 x$
e^x	e^x
$e^{g(x)}$	$g'(x)e^{g(x)}$
a^x	$(\ln a)\ a^x$
$\ln x$	$\frac{1}{x}$
$\ln g(x)$	$\frac{g'(x)}{g(x)}$
$\log_a x$	$\frac{1}{x \ln a}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arcsin g(x)$	$\frac{g'(x)}{\sqrt{1-g(x)^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\arctan g(x)$	$\frac{g'(x)}{1+g(x)^2}$
$\operatorname{arccsc} x$	$-\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
$\operatorname{arcsec} x$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
$\operatorname{arccot} x$	$-\frac{1}{1+x^2}$

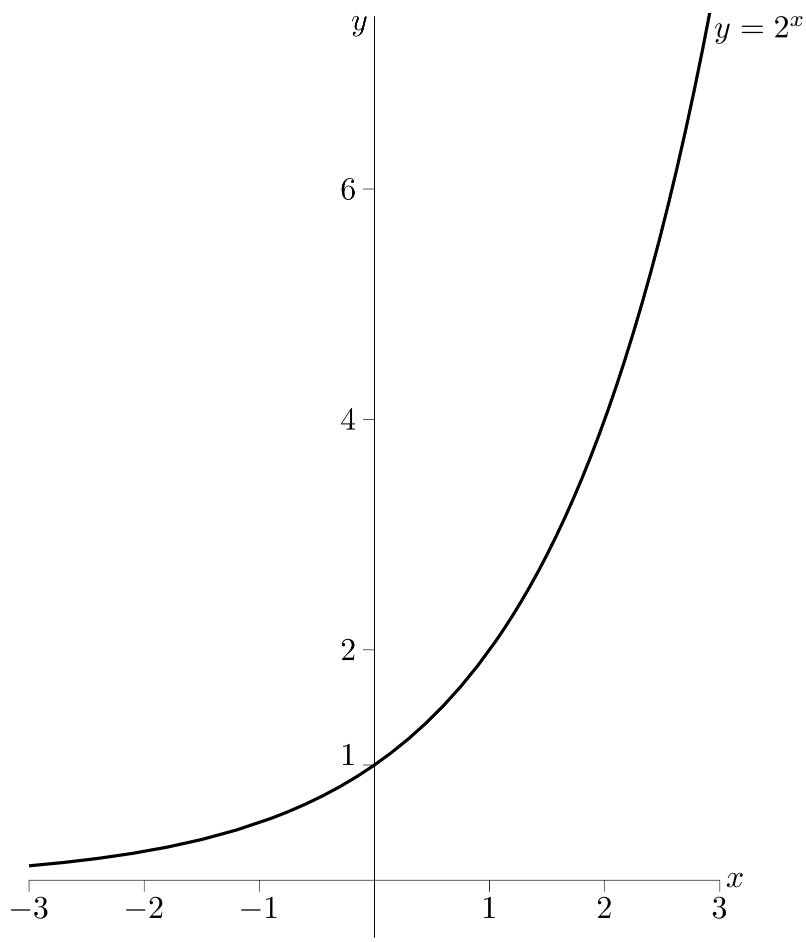
Properties of Exponentials

In the following, x and y are arbitrary real numbers, a and b are arbitrary constants that are strictly bigger than zero and e is 2.7182818284, to ten decimal places.

1) $e^0 = 1,\ \ a^0 = 1$
2) $e^{x+y} = e^x e^y,\ \ a^{x+y} = a^x a^y$
3) $e^{-x} = \frac{1}{e^x},\ \ a^{-x} = \frac{1}{a^x}$
4) $\big(e^x\big)^y = e^{xy},\ \big(a^x\big)^y = a^{xy}$
5) $\frac{d}{dx}e^x = e^x,\ \ \frac{d}{dx}e^{g(x)} = g'(x)e^{g(x)},\ \ \frac{d}{dx}a^x = (\ln a)\ a^x$
6) $\lim\limits_{x\to\infty} e^x = \infty,\ \ \lim\limits_{x\to-\infty} e^x = 0$

   $\lim\limits_{x\to\infty} a^x = \infty,\ \ \lim\limits_{x\to-\infty} a^x = 0$ if $a > 1$

   $\lim\limits_{x\to\infty} a^x = 0,\ \ \lim\limits_{x\to-\infty} a^x = \infty$ if $0 < a < 1$
7) The graph of 2^x is given below. The graph of a^x, for any $a > 1$, is similar.



<u>Properties of Logarithms</u>

In the following, x and y are arbitrary real numbers that are strictly bigger than 0, a is an arbitrary constant that is strictly bigger than one and e is 2.7182818284, to ten decimal places.

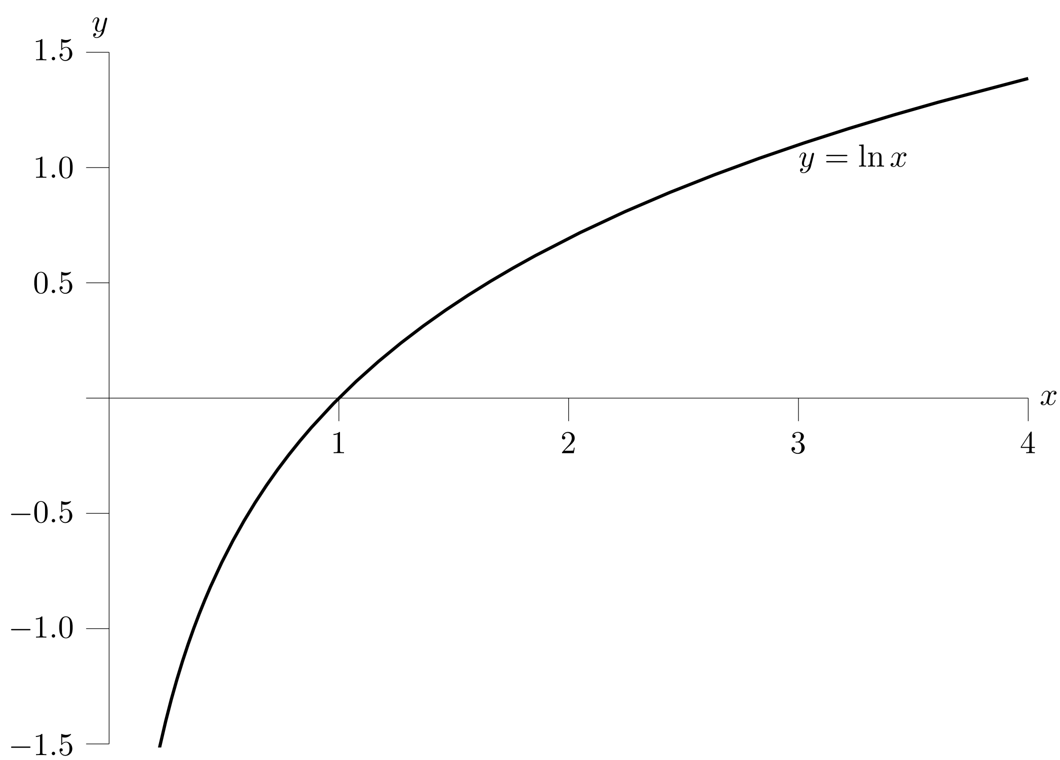
1) $e^{\ln x} = x$, $a^{\log_a x} = x$, $\log_e x = \ln x$, $\log_a x = \frac{\ln x}{\ln a}$
2) $\log_a \left(a^x\right) = x$, $\ln\left(e^x\right) = x$

   $\ln 1 = 0$, $\log_a 1 = 0$

   $\ln e = 1$, $\log_a a = 1$
3) $\ln(xy) = \ln x + \ln y$, $\log_a(xy) = \log_a x + \log_a y$
4) $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$, $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

   $\ln\left(\frac{1}{y}\right) = -\ln y$, $\log_a\left(\frac{1}{y}\right) = -\log_a y$,
5) $\ln(x^y) = y \ln x$, $\log_a(x^y) = y \log_a x$
6) $\frac{d}{dx}\ln x = \frac{1}{x}$, $\frac{d}{dx}\ln(g(x)) = \frac{g'(x)}{g(x)}$, $\frac{d}{dx}\log_a x = \frac{1}{x \ln a}$
7) $\lim_{x\to\infty} \ln x = \infty$, $\lim_{x\to 0} \ln x = -\infty$

   $\lim_{x\to\infty} \log_a x = \infty$, $\lim_{x\to 0} \log_a x = -\infty$
8) The graph of $\ln x$ is given below. The graph of $\log_a x$, for any $a > 1$, is similar.



Taylor Expansions

$$\begin{aligned}
f(x) &= f(x_0) + f'(x_0)\,(x-x_0) + \tfrac{1}{2!}f''(x_0)\,(x-x_0)^2 + \cdots + \tfrac{1}{n!}f^{(n)}(x_0)\,(x-x_0)^n \\
&\qquad + \tfrac{1}{(n+1)!}f^{(n+1)}(c)\,(x-x_0)^{n+1} \qquad \text{for some } c \text{ between } x_0 \text{ and } x \\
e^x &= \sum_{n=0}^{\infty} \tfrac{1}{n!}x^n \\
&= 1 + x + \tfrac{1}{2}x^2 + \tfrac{1}{3!}x^3 + \cdots + \tfrac{1}{n!}x^n + \cdots \\
\sin x &= \sum_{n=0}^{\infty} \tfrac{(-1)^n}{(2n+1)!}x^{2n+1} \\
&= x - \tfrac{1}{3!}x^3 + \tfrac{1}{5!}x^5 - \cdots + \tfrac{(-1)^n}{(2n+1)!}x^{2n+1} + \cdots \\
\cos x &= \sum_{n=0}^{\infty} \tfrac{(-1)^n}{(2n)!}x^{2n} \\
&= 1 - \tfrac{1}{2!}x^2 + \tfrac{1}{4!}x^4 - \cdots + \tfrac{(-1)^n}{(2n)!}x^{2n} + \cdots \\
\tfrac{1}{1-x} &= \sum_{n=0}^{\infty} x^n \\
&= 1 + x + x^2 + x^3 + \cdots + x^n + \cdots \\
\tfrac{1}{1+x} &= \sum_{n=0}^{\infty} (-1)^n x^n \\
&= 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + \cdots \\
\ln(1-x) &= -\sum_{n=1}^{\infty} \tfrac{1}{n}x^n \\
&= -x - \tfrac{1}{2}x^2 - \tfrac{1}{3}x^3 - \cdots - \tfrac{1}{n}x^n - \cdots \\
\ln(1+x) &= -\sum_{n=1}^{\infty} \tfrac{(-1)^n}{n}x^n \\
&= x - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 - \cdots - \tfrac{(-1)^n}{n}x^n - \cdots \\
(1+x)^p &= 1 + px + \tfrac{p(p-1)}{2}x^2 + \tfrac{p(p-1)(p-2)}{3!}x^3 + \cdots + \tfrac{p(p-1)(p-2)\cdots(p-n+1)}{n!}x^n + \cdots
\end{aligned}$$