Part I

Circle the correct answer. You get +3 for a correct answer, -1 for a wrong answer, and 0 for no answer.

1. The ODE \( y' = e^x - y \) has the solution,
   \[
   (a) \; e^x + C \quad (b) \; e^{-x} + C \quad (c) \; \ln(e^x + C) \quad (d) \; x + C \\
   (e) \; None \; of \; the \; above,
   \]
   where \( C \) is a constant.

2. The ODE \( y' + \frac{1}{x} y = \frac{1}{x} \) has the solution,
   \[
   (a) \; \frac{1}{x} + C \quad (b) \; \frac{C}{x} + 1 \quad (c) \; Cx + 1 \quad (d) \; \frac{C + 1}{x} \\
   (e) \; None \; of \; the \; above,
   \]
   where \( C \) is a constant.

3. The graph below can represent the solution of,
   \[
   (a) \; \ddot{x} + x = 0 \quad (b) \; \ddot{x} - x = 0 \quad (c) \; \ddot{x} + 2\ddot{x} + 2x = 0 \\
   (d) \; \ddot{x} - 2\ddot{x} + 2x = 0 \quad (e) \; None \; of \; the \; above.
   \]

4. For the ODE \( y'' - 4y' + 8y = -2\sin 2x \), the particular solution \( y_p(x) \) can be found using a trial solution of the form,
   \[
   (a) \; A\sin 2x \quad (b) \; (Ax + B)\sin 2x \quad (c) \; A\sin 2x + B\cos 2x \\
   (d) \; Ax\sin 2x + Bx\cos 2x \quad (e) \; None \; of \; the \; above,
   \]
   where \( A \) and \( B \) are the undetermined coefficients.
1. Consider the following ODE
\[
\frac{dy}{dx} + (\sin x)y = 2x e^{\cos x} + \sin x.
\]
(a) Find the integrating factor.
(b) Find the general solution and apply the initial condition \( y(0) = 1 \).
2. Solve the boundary value problem,

\[ y'' + 2y' + y = -8e^{-x} + x^2, \]

with \( y(0) = 0 \) and \( y(1) = 3 \).
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