Assignment 9: Due Friday, April 3rd at 8pm

Problems to be handed in

Problem 1 (3 points)

We consider a machine that can work for an exponential time duration of parameter λ before failing. It can have two types of failures: when the machine fails, it is a failure of type 1 with probability p, and of type 2 with probability $1 - p$. For a type 1 failure, the time to repair the machine is exponential with parameter μ_1. For a type 2 failure, it is exponential with parameter μ_2.

1. Describe this situation using a continuous-time Markov chain with 3 states and give the parameters of the model.
2. In the long run, what is the proportion of time where the machine is working? Where it is down because of a type 1 failure? Because of a type 2 failure?

Problem 2 (4 points)

In a factory, there are 4 machines and 2 repairmen. The duration of a machine before breaking is exponential with rate $\frac{1}{20}$. Once broken, the amount of time it takes to a repairman to repair it is $Exp(1/5)$. We assume the the two repairmen cannot repair the same machine at the same time.

1. In the long run, what is the proportion of time where both repairmen are busy?
2. In the long run, what is the average number of broken machines?

Problem 3 (5 points)

We consider a counter with two servers, where customers arrive at exponential rate λ and join a queue. When a server completes a service, the first customer in the queue joins this server. If a customer finds both servers free, he joins server 1 with probability $\frac{1}{2}$ and server 2 with probability $\frac{1}{2}$. The service time is $Exp(\mu_1)$ for server 1 and $Exp(\mu_2)$ for server 2. We assume $\mu_1 + \mu_2 > \lambda$.

1. What is the set of all possible states of the system?
Hint: The number of customers in the system is not always sufficient to describe the state.
2. Describe this situation using a continuous-time Markov chain and give the parameters of the model.
3. Find the limiting probabilities of this chain.
4. **(Bonus question)** We now assume that server 1 is more efficient than server 2, i.e. $\mu_1 > \mu_2$, but when a customer finds both servers free, he always joins server 1. According to the value of λ, μ_1 and μ_2, which server will be the busiest? Prove that if $\mu_2 < \mu_1 < 2\mu_2$, then server 1 is always busier, but if $\mu_1 > \mu_2$, then it depends on λ.

Recommended Problems

These provide additional practice but are not to be handed in. Textbook Chapter 6 (12th ed.): Exercises 16, 24, 29.