$\cancel{4 \mid 1+(-1)^n(2n-1)}$

Suppose n even. Then $\exists a \in \mathbb{Z}$ s.t. $n = 2a$

$1+(-1)^n(2n-1) = 1+2n-1 = 2n = 2\cdot 2a = 4a$

Then $4 \mid 1+(-1)^n(2n-1)$

Suppose n odd. Then $\exists a \in \mathbb{Z}$ s.t. $n = 2a+1$.

$1+(-1)^n(2n-1) = 1-(2n-1) = -2n+2$

$= -2(2a+1)+2 = -4a-2+2$

$= -4a = 4(-a)$

Then $1+(-1)^n(2n-1)$ is div by 4.

$\square$

Proposition: $\forall n \in \mathbb{Z}$, $1+(-1)^n(2n-1)$ is divisible by 4.

Suppose $c = \gcd(a,b)$ and $c > 1$.

$c \mid a$ and $\boxed{c \mid b}$

b is prime. So, divisors of b: $\{1, b, -1, -b\}$.

So, $c \in \{1, b, -1, -b\}$ AND $c > 1$.

So, $c = b$. $\square$

Proposition:

If b is prime and $\gcd(a,b) > 1$ (for some $a \in \mathbb{Z}$), then $\gcd(a,b) = b$.

Let $a \in \mathbb{Z}$, $a^2 \mid a$.

That is: $\exists x \in \mathbb{Z}$ st $a^2 x = a$.

Then: $a^2 x - a = 0$

$a(ax-1) = 0$

So: $\boxed{a=0}$ or $\boxed{ax=1}$

If $a=0$: then $a^2 = 0 = |0| = |a|$

If $ax=1$: a is a divisor of 1

So $a \in \{-1, 1\}$.

Then $a^2 = 1 = |a|$

$\square$

Proposition: If $a^2 \mid a$, then $|a| = a^2$.

$3 = 2^2 - 1^2$

$5 = 3^2 - 2^2$

$7 = 4^2 - 3^2$

$9 = 5^2 - 4^2$

$$2a+1 \stackrel{?}{=} (a+1)^2 - a^2$$
$$= a^2 + 2a + 1 - a^2$$
$$= 2a+1$$

Let n be any odd integer.

Then $n = 2a+1$ for some $a \in \mathbb{Z}$.

$(a+1)$, a both integers.

$(a+1)^2 - a^2 = a^2 + 2a + 1 - a^2 = 2a+1 = n$.

So n is diff. of two squares

Prop: Any odd integer is the difference of two perfect squares.

a, b, c positive #s

CASE 1: a is smallest

Then $a < \boxed{b} + c$

CASE 2: b smallest.

Then: $b < \boxed{a} + c$.

CASE 3: c smallest.

Then: $c < a + b$.

WLOG, let $\boxed{a}$ be smallest #

$(a \leq b, \quad a \leq c)$

Then:

$$\boxed{b+c} \geq a+c > \boxed{a}$$

$b \geq a$

$c \geq 0$

Proposition: Given any three positive real #s, it is possible to choose two s.t. their sum is greater than the third.

$x \geq 0$	$y \geq 0$
T	T
T	F
F	T
F	F

CASE 1: $x \geq 0$ and $y \geq 0$

WLOG CASE 3: $x \geq 0$ and $y < 0$

CASE 2: $x < 0$ and $y < 0$

Prop: $\forall x, y \in \mathbb{R}$, $|x+y| \leq |x| + |y|$

$\boxed{\text{CASE 1}}$: $x \geq 0$ and $y \geq 0$

$|x+y| = x+y = |x| + |y|$

Then $|x+y| = |x| + |y|$

so $|x+y| \leq |x| + |y|$

CASE 2: $x < 0$ and $y < 0$ $\Rightarrow$ $x+y < 0$

$$|x+y| = -(x+y) = -x + -y = |x| + |y|$$

$$|x+y| = |x| + |y|$$

$\boxed{x=2, \quad y=-5}$

$$|\underbrace{2+(-5)}_{\text{neg}}| = -[2+(-5)]$$

$\boxed{x=5, \quad y=-2}$

$$|\underbrace{5+(-2)}_{\text{pos}}| = 5-2$$

CASE 3a: $|x| \geq 0$, $y < 0$, $|x| \geq |y|$

$$|x+y| = |\underbrace{x-|y|}_{\text{pos}}| = x-|y| = |x|-|y| \underset{+2|y|}{<} |x|+|y|$$

CASE 3b:

$$|x+y| = |\underbrace{|x|-|y|}_{\text{neg}}| = -|x|+|y| \leq |x|+|y|$$

$\square$