\[ 4 \times \left( 1 + (-1)^n (2n-1) \right) \]

Suppose \( n \) even. Then \( \exists a \in \mathbb{Z} \) s.t. \( n = 2a \)

\[
1 + (-1)^n (2n-1) = 1 + 2n - 1 = 2n = 2 \cdot 2a = 4a
\]

Then \( 4 \mid 1 + (-1)^n (2n-1) \)

Suppose \( n \) odd. Then \( \exists a \in \mathbb{Z} \) s.t. \( n = 2a + 1 \).

\[
1 + (-1)^n (2n-1) = 1 - (2n-1) = -2n + 2
\]

\[
= -2(2a+1) + 2 = -4a - 2 + 2
\]

\[
= -4a = 4(-a)
\]

Then \( 1 + (-1)^n (2n-1) \) is divisible by 4.

**Proposition:** \( \forall n \in \mathbb{Z}, \ 1 + (-1)^n (2n-1) \) is divisible by 4. \( \Box \)
Suppose $c = \gcd(a, b)$ and $c > 1$.

$c \mid a$ and $c \mid b$

$b$ is prime. So, divisor of $b$: $\{1, b, -1, -b\}$.

So: $c \in \{1, b, -1, -b\}$ AND $c > 1$.

So, $c = b$.

Proposition:
If $b$ is prime and $\gcd(a, b) > 1$ (for some $a \in \mathbb{Z}$), then $\gcd(a, b) = b$. 

Let $a \in \mathbb{Z}$, $a^2 \mid a$.

That is: $\exists x \in \mathbb{Z}$ st $a^2 x = a$.

Then: $a^2 x - a = 0$
\[ a(ax - 1) = 0 \]

So: $a = 0$ or $ax = 1$.

If $a = 0$: then $a^2 = 0 = 0 = |a| = |a|$

If $ax = 1$: $a$ is a divisor of $1$

So $a \in \{-1, 1\}$.

Then $a^2 = 1 = |a|$

\[ \square \]

Proposition: If $a^2 \mid \mid \mid |a|$, then $|a| = a^2$. 
\[ 3 = 2^2 - 1^2 \]
\[ 5 = 3^2 - 2^2 \]
\[ 7 = 4^2 - 3^2 \]
\[ 9 = 5^2 - 4^2 \]

\[ 2a + 1 = (a+1)^2 - a^2 \]
\[ = a^2 + 2a + 1 - a^2 \]
\[ = 2a + 1 \]

Let \( n \) be any odd integer. Then \( n = 2a + 1 \) for some \( a \in \mathbb{Z} \).

\((a+1), \ a \) both integers.

\((a+1)^2 - a^2 = a^2 + 2a + 1 - a^2 = 2a + 1 = n.\)

So \( n \) is diff. of two squares.

Prop: Any odd integer is the difference of two perfect squares.
$a, b, c \in \mathbb{R}_+$

CASE 1: $a$ is smallest.
Then: $a \leq b, a \leq c$.

CASE 2: $b$ smallest.
Then: $b < a + c$.

CASE 3: $c$ smallest.
Then: $c < a + s$.

WLOG, let $a$ be smallest.

$(a \leq b, a \leq c)$

Then:
$b + c \geq a + c > a$

Proposition: Given any three positive real numbers, it is possible to choose two such that their sum is greater than the third.
<table>
<thead>
<tr>
<th>$x \geq 0$</th>
<th>$y \geq 0$</th>
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<tbody>
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</tbody>
</table>

**Case 1:** $x \geq 0$ and $y \geq 0$

$|x+y| = x + y = |x| + |y|$

Then $|x+y| = |x| + |y|$

So $|x+y| \leq |x| + |y|$
CASE 2: \( x < 0 \) and \( y < 0 \) \( \Rightarrow \) \( x + y < 0 \)

\[
|x+y| = -(x+y) = -x - y = |x| + |y|
\]

\( |x+y| = |x| + |y| \)

\[
\begin{array}{c}
\underline{x = 2, \quad y = -5} \\
|2 + (-5)| = -[2 + (-5)] \\
\text{neg}
\end{array}
\]

\[
\frac{X=5}{\text{neq}}
\]

\( |5 + (-2)| = 5 - 2 \)

CASE 3a: \( x > 0, \ y < 0 \) \( \Rightarrow |x| \geq |y| \)

\[
|x+y| = |x-|y|| = x - |y| = 2|x| - |y| < |x| + |y|
\]

CASE 3b:

\[
|x+y| = |(|x| - |y|| = -1|x| + |y| \leq |x| + |y|
\]