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Proposition: If P, then Q.

Proof: Suppose $\sim Q$.

$\vdots$

Therefore $\sim P$. $\square$

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Suppose x and y have opposite parity. WLOG, let x be even, so y is odd. Then there exist integers a and b such that $x = 2a$ and $y = 2b + 1$. Then $x + y = 2a + 2b + 1 = 2(a + b) + 1$. Since $a + b$ is an integer, $x + y$ is odd.

Proposition: Let $x, y \in \mathbb{Z}$. If $x + y$ is odd, then $x \neq y$.
Suppose $x = y$. Then $x + y = 2x$. Since $x \in \mathbb{Z}$, that means $x + y$ is even.

Proposition: For any $n \in \mathbb{Z}$, n^2 is even if and only if n is even.
Suppose n is even. Then $n = 2a$ for some $a \in \mathbb{Z}$, so $n^2 = 4a^2 = 2(2a^2)$. Since $2a^2 \in \mathbb{Z}$, a^2 is even.
Now, suppose n is odd. Then $n = 2a + 1$ for some $a \in \mathbb{Z}$, so $n^2 = 4a^2 + 4a + 1 = 2(2a^2 + 2a) + 1$. Since $2a^2 + 2a \in \mathbb{Z}$, n^2 is odd.

Proposition: Let $d, x, y \in \mathbb{Z}$. If $d \nmid (xy)$, then $d \nmid x$ and $d \nmid y$.
Suppose $d|x$ or $d|y$: WLOG, $d|x$. Then $da = x$ for some integer a, so $xy = day = d(ay)$. Therefore, d divides xy.

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Definition

Given $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$, we say that a and b are **congruent modulo n** if $n|(a-b)$. We express this as

$$a \equiv b \mod n.$$

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- $\forall a, b \in \mathbb{Z}, \quad a \equiv a \mod b$

Examples

For any $a, b \in \mathbb{Z}$ and any $n \in \mathbb{N}$, $a \equiv b \mod n \implies a^2 \equiv b^2 \mod n$.

For any $n, c \in \mathbb{N}$, $35c \not\equiv 72c \mod n \implies n \not| c$.

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  Since $a^2|a$ and $a \not< 0$, we conclude that $a = 0$ or $a = 1$.
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  Since a is odd, $a = 2x + 1$ for some $x \in \mathbb{Z}$.

Read Chapter 5.3 for more!