Introduction

From the Book:

In proving theorems, we apply logic to information that is considered obviously true (such as “Any two points determine exactly one line.”) or is already known to be true (e.g., the Pythagorean theorem). If our logic is correct, then anything we deduce from such information will also be true (or at least as true as the “obviously true” information we began with).
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Example:
If you are the prime minister, then you are human.
Justin Trudeau is the prime minister.
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Therefore, Justin Trudeau is human.
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I am not prime minister. Am I human?

Example:
No mammal can fly.
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Example:
No mammal can fly.
A pig is a mammal.
Therefore, a pig cannot fly. (True)
A bat is a mammal, therefore a bat cannot fly. (False)
What Are Statements?

Definition

A **statement** is a sentence or a mathematical expression that is either definitely true or definitely false.
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A **statement** is a sentence or a mathematical expression that is either definitely true or definitely false.

\[ 6 \times 3 = 18 \]
\[ 6 \times 3 \]

For every finite set \( A \), \( |\mathcal{P}(A)| = 2^{|A|} \).

For every real number \( x \), \( \sqrt{x^2} = x \).

Let \( x \) be a real number.

Multiply by the conjugate.
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<table>
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<td>$6 \times 3 = 18$</td>
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<td>\mathcal{P}(A)</td>
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<td>statement (false)</td>
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What Are Statements?

Definition

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\[ 6 \times 3 = 18 \quad \text{statement (true)} \]
\[ 6 \times 3 \quad \text{not a statement} \]

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\[ \text{statement (true)} \]

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Let \( x \) be a real number.

\[ \text{not a statement} \]

Multiply by the conjugate.

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What Are Statements?

Definition
A **statement** is a sentence or a mathematical expression that is either definitely true or definitely false.

6 × 3 = 18  statement (true)
6 × 3  not a statement

For every finite set $A$, $|\mathcal{P}(A)| = 2^{|A|}$.  statement (true)

For every real number $x$, $\sqrt{x^2} = x$.  statement (false)

Let $x$ be a real number.  statement (false)
Multiply by the conjugate.  statement (false)

We often give statements single-letter names, as we do for variables.  statement (true)
$P$: $6 \times 3 = 18$  statement (true)
We don’t always know whether a statement is true or false.

Definition

**Open sentence**: truth depends on a variable.

Example: \( x \) is prime.

This is not a statement, at least not until we know what \( x \) is.
We don’t always know whether a statement is true or false.

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**Open sentence**: truth depends on a variable.

Example: $x$ is prime.

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Goldbach Conjecture

Every even integer greater than 2 is a sum of two prime numbers.

$4 = 2 + 2$, $6 = 3 + 3$, $8 = 3 + 5$, $20 = 7 + 13$
We don’t always know whether a statement is true or false.

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4 = 2 + 2, \quad 6 = 3 + 3, \quad 8 = 3 + 5, \quad 20 = 7 + 13
\]

Holds through \( 4 \times 10^{18} \) (Wikipedia)
2. Logic

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Holds through \( 4 \times 10^{18} \) (Wikipedia)

The Goldbach Conjecture is a statement. (We don’t know whether it is true or false, but we know it is one of them!)
And + Truth Tables

Definition

For statements $P$ and $Q$, the statement "$P$ and $Q$", written $P \land Q$, is true if and only if both $P$ and $Q$ are true.
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$P$: “I washed the dishes and cleaned the sink.”
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P: \text{“I washed the dishes and cleaned the sink.”}
\]
\[
Q: \text{“I washed the dishes.”}
\]
\[
R: \text{“I cleaned the sink.”}
\]
\[
P = Q \land R
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$P$: "I washed the dishes and cleaned the sink."

$Q$: "I washed the dishes."

$R$: "I cleaned the sink."

$P = Q \land R$

If one (or both) of $Q$ or $R$ is false, then $P$ is false.

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For statements $P$ and $Q$, the statement "$P$ and $Q$", written \( P \land Q \), is true if and only if both $P$ and $Q$ are true.

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\[
\begin{array}{|c|c|c|}
\hline
Q & R & Q \land R \\
\hline
T & T & T \\
T & F & F \\
F & T & \\
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\hline
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This is different from standard English usage.

If a mathematician says “take a chocolate cookie or a peanut cookie,” you can take one of each, and you may take one cookie, but you must take a cookie.
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True or False: 3 is prime or 5 is prime.
True or False: 3 is prime or 10 is prime.
True or False: 3 is prime or some lions have green eyes.
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What if we want to express "P or Q, but not both, are true." That is: 

\[ P \lor Q \Rightarrow \neg (P \land Q) \]

or is true, and and is false.

Definition
For a statement \( P \), the statement "not \( P \)", written \( \neg P \), is true when \( P \) is false, and false when \( P \) is true. (You might also see this written as "\( \neg P \)."

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What if we want to express “P or Q, but not both, are true.”

That is: $P \lor Q$ is true, and $P \land Q$ is false.

or

and
What if we want to express “$P$ or $Q$, but not both, are true.”
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For a statement $P$, the statement “not $P$”, written $\sim P$, is true when $P$ is false, and false when $P$ is true. (You might also see this written as “$\neg P$”.)
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Or, Not

2. Logic
2.1 Statements
2.2 And, Or, Not
2.3 Conditional Statements
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2.7 Quantifiers
2.8 More on Conditional Statements
2.9 Translating English to Symbolic Logic
2.10 Negating Statements
2.11 Logical Inference

### 2.2 And, Or, Not

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<th>$\sim (P \land Q)$</th>
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<th>( \sim (P \land Q) )</th>
<th>( (P \lor Q)\land \sim(P \land Q) )</th>
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$n = 27$

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\[
\begin{array}{ccc}
P & Q & P \Rightarrow Q \\
T & T & T \\
T & F & F \\
F & T & T \\
F & F & T \\
\end{array}
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$n = 27$
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\hline
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$n = 27$

$n = 10$

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2.4 Biconditional Statements
2.5 Truth Tables for Statements
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2.7 Quantifiers
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\[ n = 27 \]
\[ n = 10 \]
\[ n = 6 \]

\[
\begin{array}{ccc}
\text{P} & \text{Q} & P \implies Q \\
\text{T} & \text{T} & \text{T} \\
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P & Q & P \Rightarrow Q & Q \Rightarrow P \\
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T & T & T & T \\
T & F & T & F \\
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F & T & T & F \\
\end{array}
\]

\( P: n \) is divisible by 9

\( Q: n \) is divisible by 3.

- \( n = 27 \)
- \( n = 10 \)
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Conditional Example

\( P \): \( f(x) \) is a continuous function on \([a, b]\).
\( Q \): \( N \) is a number strictly between \( f(a) \) and \( f(b) \).
\( R \): There exists some number \( c \in (a, b) \) such that \( f(c) = N \).
Conditional Example

\[ P: f(x) \text{ is a continuous function on } [a, b]. \]
\[ Q: N \text{ is a number strictly between } f(a) \text{ and } f(b). \]
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Intermediate Value Theorem

\[ (P \land Q) \Rightarrow R \]
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$P$: $f(x)$ is a continuous function on $[a, b]$.
$Q$: $N$ is a number strictly between $f(a)$ and $f(b)$.
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Intermediate Value Theorem

$$(P \land Q) \Rightarrow R$$

\[f(x) = \frac{1}{x^2}\]
\[a = -10, \ b = 1\]
\[N = 4\]

\[f(x) = \frac{1}{x^2}\]
\[a = -1, \ b = 1\]
\[N = \frac{1}{4}\]

\[f(x) = \frac{1}{x^2}\]
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\[N = \frac{1}{4}\]
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**P:** $f(x)$ is a continuous function on $[a, b]$.

**Q:** $N$ is a number strictly between $f(a)$ and $f(b)$.

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#### Intermediate Value Theorem

$$(P \land Q) \Rightarrow R$$

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  $a = -10, \ b = 1$
  
  $N = 4$

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  $a = -1, \ b = 1$
  
  $N = \frac{1}{4}$

- $f(x) = \frac{1}{x^2}$
  
  $a = 1, \ b = 10$
  
  $N = \frac{1}{4}$

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### 2. Logic

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\( R: \) There exists some number \( c \in (a, b) \) such that \( f(c) = N \).

**Intermediate Value Theorem**

\[(P \land Q) \Rightarrow R\]

\[f(x) = \frac{1}{x^2}\]
\(a = -10, \ b = 1\)
\(N = 4\)

\[f(x) = \frac{1}{x^2}\]
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#### Intermediate Value Theorem

$$ (P \land Q) \Rightarrow R $$

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<tr>
<th>P</th>
<th>Q</th>
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<th>$(P \land Q) \Rightarrow R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
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</table>

$f(x) = \frac{1}{x^2}$

\begin{align*}
a &= -10, \ b &= 1 \\
N &= 4
\end{align*}

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(f(x) = \frac{1}{x^2}) \\
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<p>| | | | | |</p>
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\[(P \land Q) \implies R\]

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  \hline
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  T & & & \ \\
  \hline
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The statement

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- A statement must be true or false, and not both.
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Compare:

If \( n \) is divisible by 9, then \( n \) is divisible by 3.

If \( n \) is divisible by 3, then \( n \) is divisible by 9.
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<tr>
<td>F</td>
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The example \( n = 6 \) shows that this statement is not a theorem, because it is not always true.
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If \( n \) is divisible by 9, then \( n \) is divisible by 3.

\[ B \]

\[ P \]
If \( n \) is divisible by 3, then \( n \) is divisible by 9.

\[ Q \]

\[
\begin{array}{|c|c|c|}
\hline
n = 6 & P & Q & P \Rightarrow Q \\
\hline
T & F & F & F \\
\hline
\end{array}
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If \( n \) is divisible by 9, then \( n \) is divisible by 3.

\[ n = 6 \]

\begin{array}{|c|c|c|}
\hline
P & Q & P \Rightarrow Q \\
\hline
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\hline
\end{array}

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\[ A \]

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\[ B \]

\[ \begin{array}{ccc|c}
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<tr>
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If \( A \Rightarrow B \) were false for some values of \( n \), then it would not be a theorem.

<table>
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The example \( n = 6 \) shows that this statement is not a theorem, because it is not always true.
Further Examples with Conditional Statements

(the TV is made entirely of cheese) \[ \implies \] (the TV does not turn on)
(you are in Canada) \[ \iff \] (you are in Vancouver, BC)
\[(x > 3) \implies (x \geq 3)\]
\[(x^2 > 9) \implies (|x| \geq 3)\]
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Converse Statements

Careful!

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\[ Q \Rightarrow P \] is called the **converse** of \[ P \Rightarrow Q. \]
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If you are the prime minister, then you are a human.
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If you are the prime minister, then you are a human. True.
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If you are the prime minister, then you are a human. \hspace{1cm} \text{True.}

If you are a human, then you are the prime minister.

If you are the murderer, then you were driving a blue van.

Daniel was driving a blue van.

Is Daniel the murderer?

Maybe, maybe not.
Converse Statements

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In every day life, we use “if/then” statements imprecisely.
If you are registered for this course, then you will turn in homework on Friday.
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Biconditional Statements (if and only if)

**Definition**

A statement of the form $P \iff Q$ (meaning $P \Rightarrow Q$ AND $Q \Rightarrow P$) is **biconditional**. It means that $P$ is true if and only if $Q$ is true. In this case, we think of $P$ and $Q$ as being equivalent.
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Conditional or Biconditional?

$(n$ is an even integer), $\quad (n$ is an integer divisible by 2)$

$(T$ is teaching our class right now), $\quad (T$ is named Elyse)$

If $x$ is any real number,$

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Further Examples with (Bi-)Conditional Statements

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\begin{align*}
& (x = 0) & & (4x = 0) \\
& (x^2 - 4 = 0) & & (x = 2) \\
& \text{\(x\) is an integer} & & \text{\(x - 17\) is an integer} \\
& \text{\(x\) is odd} & & \text{\(1 + x\) is even} \\
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\[(A \land B \land C) \lor (\sim A \land D) \lor (\sim B \land E)\]
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How many rows will be in the truth table?

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Logical Equivalence and Truth Tables

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</table>
Definition

Two statements are logically equivalent if (and only if) they have exactly the same values in a truth table.

\[(P \text{ and } Q \text{ are logically equivalent}) \iff (P \iff Q)\]

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>\sim P</th>
<th>\sim Q</th>
<th>P \land Q</th>
<th>(\sim P \lor \sim Q)</th>
<th>\sim (P \land Q)</th>
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<th>(\sim P)</th>
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</table>
Logical Equivalence and Truth Tables

Definition

Two statements are logically equivalent if (and only if) they have exactly the same values in a truth table.

\((P \text{ and } Q \text{ are logically equivalent}) \Leftrightarrow (P \Leftrightarrow Q)\)

Example: \((\sim P \lor \sim Q)\) is equivalent to \(\sim (P \land Q)\)

<table>
<thead>
<tr>
<th>(P)</th>
<th>(Q)</th>
<th>(\sim P)</th>
<th>(\sim Q)</th>
<th>(P \land Q)</th>
<th>(\sim (P \lor \sim Q))</th>
<th>(\sim (P \land Q))</th>
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### Logical Equivalence and Truth Tables

#### Definition

Two statements are logically equivalent if (and only if) they have exactly the same values in a truth table.

\((P \text{ and } Q \text{ are logically equivalent}) \iff (P \iff Q)\)

<table>
<thead>
<tr>
<th></th>
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<th>~ P</th>
<th>~ Q</th>
<th>(P \land Q)</th>
<th>(\sim (P \lor \sim Q))</th>
<th>(\sim (P \land Q))</th>
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Logical Equivalence

Which of the following are logically equivalent statements?

- $P \Rightarrow Q$
- $\sim P \land \sim Q$
- $\sim P \lor Q$
- $\sim (P \lor Q)$
Which of the following are logically equivalent statements?

- $P \Rightarrow Q$
- $\sim P \wedge \sim Q$
- $\sim P \lor Q$
- $\sim (P \lor Q)$

<table>
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<tr>
<th></th>
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<th>$P \Rightarrow Q$</th>
<th>$\sim P \wedge \sim Q$</th>
<th>$\sim P \lor Q$</th>
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Which of the following are logically equivalent statements?

- \( P \implies Q \)
- \( \neg P \land \neg Q \)
- \( \neg P \lor Q \)
- \( \neg (P \lor Q) \)

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<thead>
<tr>
<th></th>
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<th>( P \implies Q )</th>
<th>( \neg P \land \neg Q )</th>
<th>( \neg P \lor Q )</th>
<th>( \neg (P \lor Q) )</th>
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De Morgan’s Laws:

- \( \sim (P \land Q) \iff (\sim P \lor \sim Q) \)
- \( \sim (P \lor Q) \iff (\sim P \land \sim Q) \)
Common Equivalences

De Morgan’s Laws:
- \( \sim (P \land Q) \iff (\sim P \lor \sim Q) \)
- \( \sim (P \lor Q) \iff (\sim P \land \sim Q) \)

Associative Laws:
- \((P \land Q) \land R \iff P \land (Q \land R) \iff (P \land Q \land R)\)
- \((P \lor Q) \lor R \iff P \lor (Q \lor R) \iff (P \lor Q \lor R)\)
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- $\sim (P \land Q) \iff (\sim P \lor \sim Q)$
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Contrapositive:

- $(P \implies Q) \iff ((\sim Q) \implies (\sim P))$
Common Equivalences

De Morgan’s Laws:

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Contrapositive:

- \((P \Rightarrow Q) \iff ((\sim Q) \Rightarrow (\sim P))\)
Common Equivalences

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- \( (P \land Q) \land R \Leftrightarrow P \land (Q \land R) \Leftrightarrow (P \land Q \land R) \)
- \( (P \lor Q) \lor R \Leftrightarrow P \lor (Q \lor R) \Leftrightarrow (P \lor Q \lor R) \)

Contrapositive:
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Contrapositive:
- $(P \Rightarrow Q) \iff ((\sim Q) \Rightarrow (\sim P))$

\[
\begin{array}{c|c|c}
P & P \text{ true} & P \text{ false} \\
\hline
Q \text{ is true} & \text{maybe Q true} & \text{maybe Q false} \\
\hline
\sim Q \Rightarrow \sim P
\end{array}
\]
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- \( (P \Rightarrow Q) \iff ((\sim Q) \Rightarrow (\sim P)) \)

\[ \sim Q \]

\[ \sim Q \Rightarrow \sim P \]
Common Equivalences

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- $(P \Rightarrow Q) \iff ((\sim Q) \Rightarrow (\sim P))$
Implication (Conditional Statements) and the Contrapositive

\[(P \Rightarrow Q) \iff ((\sim Q) \Rightarrow (\sim P))\]

Let the Venn diagram below represent two sets: the set of all instances in which statement \( P \) is true, and the set of all instances in which \( Q \) is true. If \( P \Rightarrow Q \), label \( P \) and \( Q \) below.
Implication (Conditional Statements) and the Contrapositive

\[(P \Rightarrow Q) \iff ((\sim Q) \Rightarrow (\sim P))\]

Let the Venn diagram below represent two sets: the set of all instances in which statement \(P\) is true, and the set of all instances in which \(Q\) is true. If \(P \Rightarrow Q\), label \(P\) and \(Q\) below.
Common Equivalences

Contrapositive:

- \((P \Rightarrow Q) \Leftrightarrow ((\sim Q) \Rightarrow (\sim P))\)
Common Equivalences

Contrapositive:

- \((P \Rightarrow Q) \iff ((\sim Q) \Rightarrow (\sim P))\)

If you are prime minister, then you are a human.
Common Equivalences

Contrapositive:

- \((P \Rightarrow Q) \iff ((\sim Q) \Rightarrow (\sim P))\)

If you are prime minister, then you are a human.
If you are not a human, then you are not prime minister.
Common Equivalences

Contrapositive:

\[ (P \Rightarrow Q) \Leftrightarrow (\sim Q \Rightarrow \sim P) \]

If you are prime minister, then you are a human.
If you are not a human, then you are not prime minister.

If you are the murderer, then you drove a blue van yesterday.
Common Equivalences

Contrapositive:

\[(P \Rightarrow Q) \iff ((\sim Q) \Rightarrow (\sim P))\]

If you are prime minister, then you are a human.
If you are not a human, then you are not prime minister.

If you are the murderer, then you drove a blue van yesterday.
Jin did not drive a blue van yesterday.
Common Equivalences

Contrapositive:

\[ (P \Rightarrow Q) \Leftrightarrow ((\sim Q) \Rightarrow (\sim P)) \]

If you are prime minister, then you are a human.
If you are not a human, then you are not prime minister.

If you are the murderer, then you drove a blue van yesterday.
Jin did not drive a blue van yesterday. Can Jin be the murderer?
Common Equivalences

Contrapositive:

\[(P \Rightarrow Q) \iff ((\sim Q) \Rightarrow (\sim P))\]

If you are prime minister, then you are a human.
If you are not a human, then you are not prime minister.

If you are the murderer, then you drove a blue van yesterday.
Jin did not drive a blue van yesterday. Can Jin be the murderer? Logically, Jin is definitely not the murderer.
Common Equivalences

Contrapositive:

\[ (P \Rightarrow Q) \iff ((\sim Q) \Rightarrow (\sim P)) \]

If you are prime minister, then you are a human.
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Let \( f(x) \) be a differentiable function. If \( f(x) \) has at least two roots, then \( f'(x) = 0 \) somewhere. (Rolle’s Theorem)
Contrapositive:

- \((P \Rightarrow Q) \Leftrightarrow ((\sim Q) \Rightarrow (\sim P))\)

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Let \(f(x)\) be a differentiable function. If \(f(x)\) has at least two roots, then \(f'(x) = 0\) somewhere. (Rolle’s Theorem)
Let \(f(x)\) be a differentiable function. If \(f'(x)\) is never zero, then \(f(x)\) has at most one root.
Contrapositive:

\[(P \Rightarrow Q) \iff ((\sim Q) \Rightarrow (\sim P))\]

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Let \( f(x) \) be a differentiable function. If \( f(x) \) has at least two roots, then \( f'(x) = 0 \) somewhere. (Rolle’s Theorem)

Let \( f(x) \) be a differentiable function. If \( f'(x) \) is never zero, then \( f(x) \) has at most one root.
Example: \( f(x) = e^x + x - 1 \)
A Statement is Logically Equivalent to its Contrapositive

For each statement below, choose the statement that is logically equivalent.

\[ P \Rightarrow Q \]

(a) \( Q \Rightarrow P \)
(b) \( \sim P \Rightarrow \sim Q \)
(c) \( \sim Q \Rightarrow \sim P \)
A Statement is Logically Equivalent to its Contrapositive

For each statement below, choose the statement that is logically equivalent.

\[ P \Rightarrow Q \]

(a) \( Q \Rightarrow P \)
(b) \( \sim P \Rightarrow \sim Q \)
(c) \( \sim Q \Rightarrow \sim P \)
A Statement is Logically Equivalent to its Contrapositive

For each statement below, choose the statement that is logically equivalent.

\[ P \Rightarrow Q \]

(a) \( Q \Rightarrow P \)
(b) \( \sim P \Rightarrow \sim Q \)
(c) \( \sim Q \Rightarrow \sim P \)

If it is a square, then it is a rectangle.

(a) If it is a rectangle, then it is a square.
(b) If is it not a square, then it is not a rectangle.
(c) If it is not a rectangle, then it is not a square.
(d) It is a square if and only if it is a rectangle.
(e) It is a square if and only if it is a square.
For each statement below, choose the statement that is logically equivalent.

\[ P \implies Q \]

(a) \( Q \implies P \)
(b) \( \sim P \implies \sim Q \)
(c) \( \sim Q \implies \sim P \)

If it is a square, then it is a rectangle.

(a) If it is a rectangle, then it is a square.
(b) If it is not a square, then it is not a rectangle.
(c) **If it is not a rectangle, then it is not a square.**
(d) It is a square if and only if it is a rectangle.
(e) It is a square if and only if it is a square.
A Statement is Logically Equivalent to its Contrapositive

For each statement below, choose the statement that is logically equivalent.

\[ P \implies Q \]

(a) \( Q \implies P \)
(b) \( \sim P \implies \sim Q \)
(c) \( \sim Q \implies \sim P \)

If it is a square, then it is a rectangle.

(a) If it is a rectangle, then it is a square.
(b) If is it not a square, then it is not a rectangle.
(c) \textit{If it is not a rectangle, then it is not a square.}
(d) It is a square if and only if it is a rectangle.
(e) It is a square if and only if it is a square.

If you cook, then you clean.

(a) If you do not cook, then you do not clean.
(b) If you do not clean, then you do not cook.
(c) You cook if and only if you clean.
(d) If you do not clean, you cooked.
A Statement is Logically Equivalent to its Contrapositive

For each statement below, choose the statement that is logically equivalent.

\[ P \Rightarrow Q \]

(a) \( Q \Rightarrow P \)
(b) \( \sim P \Rightarrow \sim Q \)
(c) \( \sim Q \Rightarrow \sim P \)

If it is a square, then it is a rectangle.

(a) If it is a rectangle, then it is a square.
(b) If it is not a square, then it is not a rectangle.
(c) **If it is not a rectangle, then it is not a square.**
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If you cook, then you clean.

(a) If you do not cook, then you do not clean.
(b) **If you do not clean, then you do not cook.**
(c) You cook if and only if you clean.
(d) If you do not clean, you cooked.
A Statement is Logically Equivalent to its Contrapositive

For each statement below, choose the statement that is logically equivalent.

If the year is 1999, then we are partying.

(a) If we are not partying, then the year is not 1999.
(b) If the year is not 1999, then we are not partying.
(c) We are partying if and only if the year is 1999.

If you aren’t part of the solution, you’re part of the problem.

(a) If you are a part of the problem, then you are not a part of the solution.
(b) You are a part of the solution if and only if you are not a part of the problem.
(c) If you are not a part of the problem, then you are a part of the solution.
(d) (none of the above)

If it’s Tuesday, then it is raining.

(a) If it is not raining, then it is not Tuesday.
(b) If it is raining, then it is Tuesday.
(c) It is raining if and only if it is Tuesday.
A Statement is Logically Equivalent to its Contrapositive

For each statement below, choose the statement that is logically equivalent.

If the year is 1999, then we are partying.

(a) If we are not partying, then the year is not 1999.
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(c) We are partying if and only if the year is 1999.

If you aren’t part of the solution, you’re part of the problem.

(a) If you are a part of the problem, then you are not a part of the solution.
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A Statement is Logically Equivalent to its Contrapositive

For each statement below, choose the statement that is logically equivalent.

If the year is 1999, then we are partying.

(a) If we are not partying, then the year is not 1999.
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A Statement is Logically Equivalent to its Contrapositive

For each statement below, choose the statement that is logically equivalent.

If the year is 1999, then we are partying.

(a) If we are not partying, then the year is not 1999.
(b) If the year is not 1999, then we are not partying.
(c) We are partying if and only if the year is 1999.

If you aren’t part of the solution, you’re part of the problem.

(a) If you are a part of the problem, then you are not a part of the solution.
(b) You are a part of the solution if and only if you are not a part of the problem.
(c) If you are not a part of the problem, then you are a part of the solution.
(d) (none of the above)

If it’s Tuesday, then it is raining.

(a) If it is not raining, then it is not Tuesday.
(b) If it is raining, then it is Tuesday.
(c) It is raining if and only if it is Tuesday.
Quantifiers

Definition

The **universal quantifier** , ∀, means “for every” or “for all.”

The **existential quantifier** , ∃, means “there is” or “there exists.”
Quantifiers

Definition
The **universal quantifier**, \( \forall \), means “for every” or “for all.”
The **existential quantifier**, \( \exists \), means “there is” or “there exists.”

We write \( \not\exists \) to mean “there does not exist.”
Quantifiers

Definition

The **universal quantifier** \( \forall \), means “for every” or “for all.”

The **existential quantifier** \( \exists \), means “there is” or “there exists.”

We write \( \nexists \) to mean “there does not exist.”

Example: \( \forall \) action, \( \exists \) equal and opposite reaction.
Quantifiers

Definition

The **universal quantifier**, ∀, means “for every” or “for all.”

The **existential quantifier**, ∃, means “there is” or “there exists.”

We write ∄ to mean “there does not exist.”

Example: ∀ action, ∃ equal and opposite reaction.

We often omit “such that,” or replace it with a comma or colon.
Quantifiers

**Definition**

The **universal quantifier**, \( \forall \), means “for every” or “for all."

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**Example:** \( \forall \) action, \( \exists \) equal and opposite reaction.

We often omit “such that,” or replace it with a comma or colon.

**True or False:**

- \( \forall x \in \mathbb{Q}, \frac{1}{x} \in \mathbb{Q}. \)
- \( \forall x \in \mathbb{N}, \forall y \in \mathbb{N}, x - y \in \mathbb{Z}. \)
- \( \forall x \in \mathbb{R} - \{0\}, \exists y \in \mathbb{R} \) such that \( xy = 1. \)
- \( \exists x \in \mathbb{R} - \{0\} \) such that \( \forall y \in \mathbb{R}, xy = 1. \)
- \( \forall x \in \mathbb{Q}, \forall n \in \mathbb{N}, \exists z \in \mathbb{Z} \) such that \( x = \frac{z}{n}. \)
- \( \forall x \in \mathbb{Z}, \forall n \in \mathbb{N}, \exists z \in \mathbb{Z} \) such that \( x = \frac{z}{n}. \)
Quantifiers

Definition

The universal quantifier, $\forall$, means “for every” or “for all.”

The existential quantifier, $\exists$, means “there is” or “there exists.”

We write $\exists$ to mean “there does not exist.”

Example: $\forall$ action, $\exists$ equal and opposite reaction.
We often omit “such that,” or replace it with a comma or colon.

True or False:

- $\forall x \in \mathbb{Q}, \frac{1}{x} \in \mathbb{Q}$. **False: $x = 0$**
- $\forall x \in \mathbb{N}, \forall y \in \mathbb{N}, x - y \in \mathbb{Z}$.
- $\forall x \in \mathbb{R} - \{0\}, \exists y \in \mathbb{R}$ such that $xy = 1$.
- $\exists x \in \mathbb{R} - \{0\}$ such that $\forall y \in \mathbb{R}, xy = 1$.
- $\forall x \in \mathbb{Q}, \forall n \in \mathbb{N}, \exists z \in \mathbb{Z}$ such that $x = \frac{z}{n}$.
- $\forall x \in \mathbb{Z}, \forall n \in \mathbb{N}, \exists z \in \mathbb{Z}$ such that $x = \frac{z}{n}$.
Quantifiers

**Definition**

The **universal quantifier**, \( \forall \), means “for every” or “for all.”

The **existential quantifier**, \( \exists \), means “there is” or “there exists.”

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Example: \( \forall \) action, \( \exists \) equal and opposite reaction.

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**True or False:**

<table>
<thead>
<tr>
<th>Statement</th>
<th>True/False</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \forall x \in \mathbb{Q}, \frac{1}{x} \in \mathbb{Q} ).</td>
<td>False: ( x = 0 )</td>
</tr>
<tr>
<td>( \forall x \in \mathbb{N}, \forall y \in \mathbb{N}, x - y \in \mathbb{Z} ).</td>
<td>True</td>
</tr>
<tr>
<td>( \forall x \in \mathbb{R} - {0}, \exists y \in \mathbb{R} \text{ such that } xy = 1 ).</td>
<td></td>
</tr>
<tr>
<td>( \exists x \in \mathbb{R} - {0} \text{ such that } \forall y \in \mathbb{R}, xy = 1 ).</td>
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</table>
## Quantifiers

### Definition

The **universal quantifier**, $\forall$, means “for every” or “for all.”

The **existential quantifier**, $\exists$, means “there is” or “there exists.”

We write $\neg\exists$ to mean “there does not exist.”

### Example:

\[ \forall \text{ action}, \exists \text{ equal and opposite reaction}. \]

We often omit “such that,” or replace it with a comma or colon.

### True or False:

- $\forall x \in \mathbb{Q}, \frac{1}{x} \in \mathbb{Q}$.  
  **False:** $x = 0$

- $\forall x \in \mathbb{N}, \forall y \in \mathbb{N}, x - y \in \mathbb{Z}$.  
  **True**

- $\forall x \in \mathbb{R} - \{0\}, \exists y \in \mathbb{R}$ such that $xy = 1$.

- $\exists x \in \mathbb{R} - \{0\}$ such that $\forall y \in \mathbb{R}, xy = 1$.

- $\forall x \in \mathbb{Q}, \forall n \in \mathbb{N}, \exists z \in \mathbb{Z}$ such that $x = \frac{z}{n}$.

- $\forall x \in \mathbb{Z}, \forall n \in \mathbb{N}, \exists z \in \mathbb{Z}$ such that $x = \frac{z}{n}$.
Quantifiers

Definition

The **universal quantifier** , ∀, means “for every” or “for all.”

The **existential quantifier** , ∃, means “there is” or “there exists.”

We write ∄ to mean “there does not exist.”

Example: ∀ action, ∃ equal and opposite reaction.
We often omit “such that,” or replace it with a comma or colon.

True or False:

- ∀x ∈ Q, \( \frac{1}{x} \) ∈ Q.  
  False: \( x = 0 \)

- ∀x ∈ N, ∃y ∈ N, x − y ∈ Z.  
  True

- ∀x ∈ R − \{0\}, ∃y ∈ R such that xy = 1.  
  False: \( y = \frac{1}{x} \)

- ∃x ∈ R − \{0\} such that ∀y ∈ R, xy = 1.  
  True

- ∀x ∈ Q, ∀n ∈ N, ∃z ∈ Z such that x = \( \frac{z}{n} \).  
  False: \( x = \frac{1}{0} \)

- ∀x ∈ Z, ∀n ∈ N, ∃z ∈ Z such that x = \( \frac{z}{n} \).  
  True
Quantifiers

**Definition**

The **universal quantifier**, $\forall$, means “for every” or “for all.”

The **existential quantifier**, $\exists$, means “there is” or “there exists.”

We write $\neg\exists$ to mean “there does not exist.”

**Example:** $\forall$ action, $\exists$ equal and opposite reaction.
We often omit “such that,” or replace it with a comma or colon.

**True or False:**

- $\forall x \in \mathbb{Q}, \frac{1}{x} \in \mathbb{Q}$.  
  *False: $x = 0$*

- $\forall x \in \mathbb{N}, \forall y \in \mathbb{N}, x - y \in \mathbb{Z}$.  
  *True*

- $\forall x \in \mathbb{R} - \{0\}, \exists y \in \mathbb{R}$ such that $xy = 1$.  
  *True: $y = \frac{1}{x}$*

- $\exists x \in \mathbb{R} - \{0\}$ such that $\forall y \in \mathbb{R}, xy = 1$.  
  *False: $y = 0$*

- $\forall x \in \mathbb{Q}, \forall n \in \mathbb{N}, \exists z \in \mathbb{Z}$ such that $x = \frac{z}{n}$.  
  *False: $x = \frac{1}{3}, n = 2$*

- $\forall x \in \mathbb{Z}, \forall n \in \mathbb{N}, \exists z \in \mathbb{Z}$ such that $x = \frac{z}{n}$.  
  *True*
Quantifiers

Definition

The **universal quantifier**, $\forall$, means “for every” or “for all.”

The **existential quantifier**, $\exists$, means “there is” or “there exists.”

We write $\not\exists$ to mean “there does not exist.”

Example: $\forall$ action, $\exists$ equal and opposite reaction.
We often omit “such that,” or replace it with a comma or colon.

True or False:

- $\forall x \in \mathbb{Q}, \frac{1}{x} \in \mathbb{Q}.$  \hspace{1cm} False: $x = 0$
- $\forall x \in \mathbb{N}, \forall y \in \mathbb{N}, x - y \in \mathbb{Z}.$  \hspace{1cm} True
- $\forall x \in \mathbb{R} - \{0\}, \exists y \in \mathbb{R}$ such that $xy = 1.$  \hspace{1cm} True: $y = \frac{1}{x}$
- $\exists x \in \mathbb{R} - \{0\}$ such that $\forall y \in \mathbb{R}, xy = 1.$  \hspace{1cm} False: $y = 0$
- $\forall x \in \mathbb{Q}, \forall n \in \mathbb{N}, \exists z \in \mathbb{Z}$ such that $x = \frac{z}{n}.$  \hspace{1cm} False: $x = \frac{1}{3}, n = 2$
- $\forall x \in \mathbb{Z}, \forall n \in \mathbb{N}, \exists z \in \mathbb{Z}$ such that $x = \frac{z}{n}.$  \hspace{1cm} True: $z = xn$
Fill in the Blank

Fill in each of the boxes with $\forall$, $\exists$, or $\exists^\neg$. If multiple answers are possible, choose the best.

$\square$ integer $n$, $2n$ is even.

$\square$ integer $n$, $3n$ is even.

$\square$ real $x \neq 0$, $\square$ real $y$ such that $xy = 1$.

$\square$ real $x$ such that $\forall y \in \mathbb{R}$, $xy = 0$.

$\square$ $x \in \mathbb{R}$ such that $\forall y \in \mathbb{R}$, $xy \neq 0$. 
Fill in each of the boxes with $\forall$, $\exists$, or $\not\exists$. If multiple answers are possible, choose the best.

- $\forall$ integer $n$, $2n$ is even.
  - Also $\exists$ is true.

- $\exists$ integer $n$, $3n$ is even.

- $\not\exists$ integer $n$, $3n$ is even.

- $\not\exists$ integer $n$, $3n$ is even.

- $\forall$ real $x \neq 0$, $\exists$ real $y$ such that $xy = 1$.
  - Since $x$ is nonzero, choose $y = \frac{1}{x}$.

- $\exists$ real $x$ such that $\forall y \in \mathbb{R}$, $xy = 0$.
  - That $x$ is $x = 0$.

- $\not\exists$ real $x$ such that $\forall y \in \mathbb{R}$, $xy \neq 0$.
  - No $x$ has $xy \neq 0$ when $y = 0$. 
Fill in each of the boxes with $\forall$, $\exists$, or $\not\exists$. If multiple answers are possible, choose the best.

$\forall$ integer $n$, $2n$ is even. Also $\exists$ is true.

$\exists$ integer $n$, $3n$ is even. For example, $n = 2$.

$\not\exists$ real $x \neq 0$, $\exists$ real $y$ such that $xy = 1$.

$\exists$ real $x$ such that $\forall y \in \mathbb{R}$, $xy = 0$.

$\not\exists$ $x \in \mathbb{R}$ such that $\forall y \in \mathbb{R}$, $xy \neq 0$. 
Fill in each of the boxes with ∀, ∃, or ∄. If multiple answers are possible, choose the best.

∀ integer \( n \), \( 2n \) is even.  
Also \( ∃ \) is true.

∃ integer \( n \), \( 3n \) is even.  
For example, \( n = 2 \).

∀ real \( x \neq 0 \), ∃ real \( y \) such that \( xy = 1 \). Since \( x \) is nonzero, choose \( y = \frac{1}{x} \).

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∀ real \( x \in \mathbb{R} \) such that \( \forall y \in \mathbb{R}, xy \neq 0 \).
Fill in each of the boxes with \( \forall, \exists, \) or \( \not\exists \). If multiple answers are possible, choose the best.

\( \forall \) integer \( n \), \( 2n \) is even.

Also \( \exists \) is true.

\( \exists \) integer \( n \), \( 3n \) is even.

For example, \( n = 2 \).

\( \forall \) real \( x \neq 0 \), \( \exists \) real \( y \) such that \( xy = 1 \). Since \( x \) is nonzero, choose \( y = \frac{1}{x} \).

\( \exists \) real \( x \) such that \( \forall y \in \mathbb{R}, xy = 0 \).

That \( x \) is \( x = 0 \).

\( \not\exists \) \( x \in \mathbb{R} \) such that \( \forall y \in \mathbb{R}, xy \neq 0 \).
Fill in each of the boxes with \( \forall \), \( \exists \), or \( \nexists \). If multiple answers are possible, choose the best.

\[ \forall \text{ integer } n, 2n \text{ is even.} \] Also \( \exists \) is true.

\[ \exists \text{ integer } n, 3n \text{ is even.} \] For example, \( n = 2 \).

\[ \forall \text{ real } x \neq 0, \exists \text{ real } y \text{ such that } xy = 1. \] Since \( x \) is nonzero, choose \( y = \frac{1}{x} \).

\[ \exists \text{ real } x \text{ such that } \forall y \in \mathbb{R}, xy = 0. \] That \( x \) is \( x = 0 \).

\[ \forall x \in \mathbb{R} \text{ such that } \forall y \in \mathbb{R}, xy \neq 0. \] No \( x \) has \( xy \neq 0 \) when \( y = 0 \).
Conditional Statements with Variables

Every universally qualified statement (that is, a statement containing “∀”) can be expressed as a conditional statement with a variable.
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∀x ∈ N, x > 0
Conditional Statements with Variables

Every universally qualified statement (that is, a statement containing “∀”) can be expressed as a conditional statement with a variable.

\[ \forall x \in \mathbb{N}, \ x > 0 \]
\[ x \in \mathbb{N} \Rightarrow x > 0 \]
Every universally qualified statement (that is, a statement containing “∀”) can be expressed as a conditional statement with a variable.

∀x ∈ ℕ, x > 0
x ∈ ℕ ⇒ x > 0

∀x, y ∈ ℚ, xy ∈ ℚ
Every universally qualified statement (that is, a statement containing “∀”) can be expressed as a conditional statement with a variable.

∀x ∈ N, x > 0
x ∈ N ⇒ x > 0

∀x, y ∈ Q, xy ∈ Q
x, y ∈ Q ⇒ xy ∈ Q
Every rational number is also a real number.

If $x$ is a rational number, then $x^2$ is rational as well.

Every integer is even or odd.

No real number is both even and irrational.

Every quadratic function with real coefficients has at least one (possibly complex) root.

( $\mathbb{C}$ is the set of complex numbers.)
Every rational number is also a real number.
\( \forall r \in \mathbb{Q}, \ r \in \mathbb{R} \)
\( \mathbb{Q} \subseteq \mathbb{R} \)

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Translating English to Symbolic Logic

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\[ \forall x \in \mathbb{R}, \ x^2 \in \mathbb{R} \]

Every integer is even or odd.
\[ \forall z \in \mathbb{Z}, \ (z \text{ even}) \lor (z \text{ odd}) \]

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\[ \forall x \in \mathbb{R}, \ \sim (x \text{ even } \land x \notin \mathbb{Q}) \]

Every quadratic function with real coefficients has at least one (possibly complex) root.
( \( \mathbb{C} \) is the set of complex numbers.)
Every rational number is also a real number.
\[ \forall r \in \mathbb{Q}, \; r \in \mathbb{R} \]
\[ \mathbb{Q} \subseteq \mathbb{R} \]

If \( x \) is a rational number, then \( x^2 \) is rational as well.
\[ x \in \mathbb{R} \Rightarrow x^2 \in \mathbb{R} \]
\[ \forall x \in \mathbb{R}, \; x^2 \in \mathbb{R}. \]

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\[ \forall z \in \mathbb{Z}, \; (z \text{ even}) \lor (z \text{ odd}) \]

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\[ \forall x \in \mathbb{R}, \; \sim (x \text{ even} \; \land \; x \notin \mathbb{Q}) \]

Every quadratic function with real coefficients has at least one (possibly complex) root.
( \( \mathbb{C} \) is the set of complex numbers.)
\[ \forall a, b, c \in \mathbb{R}, \; \exists x \in \mathbb{C}, \; ax^2 + bx + c = 0. \]
At least one of the numbers \( x \) and \( y \) is even.
At least one of the numbers \( x \) and \( y \) is even.

\((x \text{ even}) \lor (y \text{ even})\)
Translating

At least one of the numbers $x$ and $y$ is even.
$(x \text{ even}) \lor (y \text{ even})$

$x$ is even, but $y$ is odd

Whenever $x$ is a perfect square, $x^3$ is as well.

$P$ is true only if $Q$ is true.
(For example: "I sing only if I’m in the shower.")

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$(x \text{ even}) \land (y \text{ odd})$

Whenever $x$ is a perfect square, $x^3$ is as well.
$\sqrt{x} \in \mathbb{Z} \Rightarrow \sqrt{x^3} \in \mathbb{Z}$

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At least one of the numbers $x$ and $y$ is even.

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$P \Rightarrow Q$

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At least one of the numbers \( x \) and \( y \) is even.
\((x \text{ even}) \lor (y \text{ even})\)

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(For example: "I sing if I’m in the shower.")
\( Q \Rightarrow P \)
\( P \Leftarrow Q \)
### Negating Statements using DeMorgan’s Laws

**DeMorgan’s Laws**

\[
\begin{align*}
\neg (P \land Q) & \iff (\neg P) \lor (\neg Q) \\
\neg (P \lor Q) & \iff (\neg P) \land (\neg Q)
\end{align*}
\]
Negating Statements using DeMorgan’s Laws

DeMorgan’s Laws

\[
\sim (P \land Q) \iff (\sim P) \lor (\sim Q) \\\n\sim (P \lor Q) \iff (\sim P) \land (\sim Q)
\]

A number can’t be both even and odd.

Ted isn’t smart or handsome.
Negating Statements using DeMorgan’s Laws

DeMorgan’s Laws

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\sim (P \land Q) & \iff (\sim P) \lor (\sim Q) \\
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\end{align*}
\]

A number can’t be both even and odd.
\[
\forall x \sim [(x \text{ even}) \land (x \text{ odd})] = \\
\forall x (\sim (x \text{ even})) \lor (\sim (x \text{ odd}))
\]

Ted isn’t smart or handsome.
DeMorgan’s Laws

\[ \sim (P \land Q) \iff (\sim P) \lor (\sim Q) \]

\[ \sim (P \lor Q) \iff (\sim P) \land (\sim Q) \]

A number can’t be both even and odd.
\[ \forall x \sim [(x \text{ even}) \land (x \text{ odd})] = \]
\[ \forall x (\sim (x \text{ even})) \lor (\sim (x \text{ odd})) \]

Ted isn’t smart or handsome.
\[ \sim [(Ted \text{ is smart}) \lor (Ted \text{ is handsome})] = \]
\[ (\sim (Ted \text{ is smart})) \land (\sim (Ted \text{ is handsome})) \]
Negating Quantified Statements

$P$: Every swan is white.
Negating Quantified Statements

\( P: \) Every swan is white.
\( \sim P: \) At least one swan is not white.
Negating Quantified Statements

\[ P: \text{ Every swan is white.} \]
\[ \sim P: \text{ At least one swan is not white.} \]

\[ Q: \text{ No student gets an A.} \]
Negating Quantified Statements

\[ P: \text{Every swan is white.} \]
\[ \sim P: \text{At least one swan is not white.} \]

\[ Q: \text{No student gets an A.} \]
\[ \sim Q: \text{At least one student gets an A.} \]
Negating Quantified Statements

\( P: \) Every swan is white.
\( \sim P: \) At least one swan is not white.

\( Q: \) No student gets an A.
\( \sim Q: \) At least one student gets an A.

\( R: \) Everything in the library is also on the internet.
Negating Quantified Statements

\[ P: \text{Every swan is white.} \]
\[ \sim P: \text{At least one swan is not white.} \]

\[ Q: \text{No student gets an A.} \]
\[ \sim Q: \text{At least one student gets an A.} \]

\[ R: \text{Everything in the library is also on the internet.} \]
\[ \sim R: \text{At least one thing in the library is not also on the internet.} \]
Negating Quantified Statements

\[P: \text{Every swan is white.}\]
\[\sim P: \text{At least one swan is not white.}\]

\[Q: \text{No student gets an A.}\]
\[\sim Q: \text{At least one student gets an A.}\]

\[R: \text{Everything in the library is also on the internet.}\]
\[\sim R: \text{At least one thing in the library is not also on the internet.}\]

\[S: \text{There exists a human who has walked on the moon.}\]
Negating Quantified Statements

\[ P: \text{ Every swan is white.} \]
\[ \sim P: \text{ At least one swan is not white.} \]

\[ Q: \text{ No student gets an A.} \]
\[ \sim Q: \text{ At least one student gets an A.} \]

\[ R: \text{ Everything in the library is also on the internet.} \]
\[ \sim R: \text{ At least one thing in the library is not also on the internet.} \]

\[ S: \text{ There exists a human who has walked on the moon.} \]
\[ \sim S: \text{ No human has walked on the moon.} \]
\[ \sim S: \text{ For every human } H, \ H \text{ has not walked on the moon.} \]
Negating Quantified Statements

\( P \): Every swan is white.
\( \sim P \): At least one swan is not white.

\( Q \): No student gets an A.
\( \sim Q \): At least one student gets an A.

\( R \): Everything in the library is also on the internet.
\( \sim R \): At least one thing in the library is not also on the internet.

\( S \): There exists a human who has walked on the moon.
\( \sim S \): No human has walked on the moon.
\( \sim S \): For every human \( H \), \( H \) has not walked on the moon.

\( \sim (\forall x \in A, P(x)) = \exists x \in A, \sim P(x) \)
\( \sim (\exists x \in A, P(x)) = \forall x \in A, \sim P(x) \)
Negating Statements with Multiple Quantifiers

\(~ (\forall x \in A, P(x)) = \exists x \in A, ~ P(x)\)
\(~ (\exists x \in A, P(x)) = \forall x \in A, ~ P(x)\)

\(P: \forall x \in \mathbb{R}, \exists y \in \mathbb{R}, y^3 = x.\)
That is: \(P\) says that every real number has a real cube root.

\(Q: \exists x \in \mathbb{Z}, \forall y \in \mathbb{R}, xy = 0.\)
That is: there is some integer \(x\) such that \(xy = 0\) for every real number \(y\).

\(\forall A \subseteq \mathbb{N}, \exists a \in A, \forall b \in A a \leq b\)
Every subset of the natural numbers has a smallest element.

\(\forall x \in \mathbb{R}, \forall y \in \mathbb{Z}, \exists z \in \{-5, 3.8\}, xyz \geq 0\)
Negating Statements with Multiple Quantifiers

\[ \sim (\forall x \in A, P(x)) = \exists x \in A, \sim P(x) \]
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\[ \sim P: \exists x \in \mathbb{R}, \forall y \in \mathbb{R}, y^3 \neq x. \]
That is: \( \sim P \) says that there exists some real number such that every real number is not its cube root.

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Negating Statements with Multiple Quantifiers

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\(\forall A \subseteq \mathbb{N}, \exists a \in A, \forall b \in A \ a \leq b\)
Every subset of the natural numbers has a smallest element.

\(\exists A \subseteq \mathbb{N}, \forall a \in A, \exists b \in A, a > b\)
There is some subset of the natural numbers so that, no matter what element we choose, there is some element that is smaller.

\(\forall x \in \mathbb{R}, \forall y \in \mathbb{Z}, \exists z \in \{-5, 3.8\}, xyz \geq 0\)
Negating Statements with Multiple Quantifiers

\[\sim (\forall x \in A, P(x)) = \exists x \in A, \sim P(x)\]
\[\sim (\exists x \in A, P(x)) = \forall x \in A, \sim P(x)\]

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That is: for every integer \(x\), there is some real number \(y\) such that \(xy \neq 0\).

**∀A ⊆ N, ∃a ∈ A, ∀b ∈ A a ≤ b**
Every subset of the natural numbers has a smallest element.

**∃A ⊆ N, ∀a ∈ A, ∃b ∈ A, a > b**
There is some subset of the natural numbers so that, no matter what element we choose, there is some element that is smaller.

\(\forall x \in \mathbb{R}, \forall y \in \mathbb{Z}, \exists z \in \{-5, 3.8\}, xyz \geq 0\)
\(\exists x \in \mathbb{R}, \exists y \in \mathbb{Z}, \forall z \in \{-5, 3.8\}, xyz < 0\)
\(\exists x \in \mathbb{R}, \exists y \in \mathbb{Z}, [(-5xy < 0) \land (3.8xy < 0)]\)
Negating Conditional Statements

\[ \forall x \in \mathbb{R}, \left[ \frac{x}{9} \in \mathbb{Z} \Rightarrow \frac{x}{3} \in \mathbb{Z} \right] \]

\[ \forall x \in \mathbb{Z}, \left[ x < a \Rightarrow x^2 < a^2 \right] \]

\[ \forall x, y, z, n \in \mathbb{N}, \left[ (x^n + y^n = z^n) \Rightarrow n \in \{1, 2\} \right] \]

Fermat’s Last Theorem

\[ \forall \text{ function } f(x), \forall a < b \in \mathbb{R}, \left[ f(x) \text{ continuous on } [a, b] \right] \Rightarrow \left[ \forall N \text{ between } f(a) \text{ and } f(b), \exists c \in (a, b), f(c) = N \right] \]

Intermediate Value Theorem
Negating Conditional Statements

\[ \forall x \in \mathbb{R}, \left[ \frac{x}{9} \in \mathbb{Z} \Rightarrow \frac{x}{3} \in \mathbb{Z} \right] \]
\[ \exists x \in \mathbb{R}, \left[ \frac{x}{9} \in \mathbb{Z} \land \frac{x}{3} \notin \mathbb{Z} \right] \]

\[ \forall x \in \mathbb{Z}, \left[ x < a \Rightarrow x^2 < a^2 \right] \]

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**Fermat’s Last Theorem**

\[ \forall \text{ function } f(x), \forall a < b \in \mathbb{R}, \left[ f(x) \text{ continuous on } [a, b] \right] \Rightarrow \]
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**Intermediate Value Theorem**
Negating Conditional Statements

\[ \forall x \in \mathbb{R}, \left[ \frac{x}{9} \in \mathbb{Z} \implies \frac{x}{3} \in \mathbb{Z} \right] \]
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\[ \forall x \in \mathbb{Z}, \left[ x < a \implies x^2 < a^2 \right] \]
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\[ \forall x, y, z, n \in \mathbb{N}, \left[ (x^n + y^n = z^n) \implies n \in \{1, 2\} \right]. \]
Fermat’s Last Theorem

\[ \forall \text{ function } f(x), \forall a < b \in \mathbb{R}, \left[ f(x) \text{ continuous on } [a, b] \right] \implies \left[ \forall N \text{ between } f(a) \text{ and } f(b), \exists c \in (a, b), f(c) = N \right] \]
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\[ \forall x, y, z, n \in \mathbb{N}, \left[ (x^n + y^n = z^n) \Rightarrow n \in \{1, 2\} \right]. \]
Fermat’s Last Theorem
\[ \exists x, y, z, n \in \mathbb{N}, \left[ (x^n + y^n = z^n) \land (n > 2) \right] \]

\[ \forall \text{ function } f(x), \forall a < b \in \mathbb{R}, \left[ f(x) \text{ continuous on } [a, b] \right] \Rightarrow \]
[\forall N \text{ between } f(a) \text{ and } f(b), \exists c \in (a, b), f(c) = N]\nIntermediate Value Theorem
Negating Conditional Statements

\[ \forall x \in \mathbb{R}, \left[ \frac{x}{9} \in \mathbb{Z} \Rightarrow \frac{x}{3} \in \mathbb{Z} \right] \]
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Fermat’s Last Theorem
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Intermediate Value Theorem
\[ \exists \text{ function } f(x), \exists a < b \in \mathbb{R}, \left[ f(x) \text{ continuous on } [a, b] \right] \land \]
\[ \exists N \text{ between } f(a) \text{ and } f(b), \forall c \in (a, b), f(c) \neq N \]
## Logical Inference

### Definition

Given two true statements, we can conclude using pure logic that a third is true. This process is called **logical inference**.
Definition

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- $P \Rightarrow Q$
2. Logic

2.1 Statements

2.2 And, Or, Not

2.3 Conditional Statements

2.4 Biconditional Statements

2.5 Truth Tables for Statements

2.6 Logical Equivalence

2.7 Quantifiers

2.8 More on Conditional Statements

2.9 Translating English to Symbolic Logic

2.10 Negating Statements

2.11 Logical Inference

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Given two true statements, we can conclude using pure logic that a third is true. This process is called **logical inference**.

- \( P \Rightarrow Q \)
- \( P \)
## Logical Inference

**Definition**

Given two true statements, we can conclude using pure logic that a third is true. This process is called **logical inference**.

- \( P \Rightarrow Q \)
- \( P \)
- Therefore, \( Q \).
Logical Inference

Definition

Given two true statements, we can conclude using pure logic that a third is true. This process is called **logical inference**.

- $P \implies Q$
- $P$
- **Therefore, $Q$.**
- $P \implies Q$
- $\sim Q$
## Logical Inference

### Definition

Given two true statements, we can conclude using pure logic that a third is true. This process is called **logical inference**.

- \( P \implies Q \)
- \( P \)
- Therefore, \( Q \).
- \( P \implies Q \)
- \( \sim Q \)
- Therefore, \( \sim P \).
Definition

Given two true statements, we can conclude using pure logic that a third is true. This process is called \textbf{logical inference}.

- $P \Rightarrow Q$
- $P$
- Therefore, $Q$.
- $P \Rightarrow Q$
- $\sim Q$
- Therefore, $\sim P$.
- $P \lor Q$
- $\sim P$
Logical Inference

Definition
Given two true statements, we can conclude using pure logic that a third is true. This process is called **logical inference**.

- $P \Rightarrow Q$
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- Therefore, $Q$.
- $P \Rightarrow Q$
- $\sim Q$
- Therefore, $\sim P$.
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- Therefore, $Q$. 
Given two true statements, we can conclude using pure logic that a third is true. This process is called **logical inference**.

- $P \Rightarrow Q$
- $P$
- Therefore, $Q$.

- $P \Rightarrow Q$
- $\sim Q$
- Therefore, $\sim P$.

- $P \lor Q$
- $\sim (P \land Q)$
- Therefore, $\sim Q$.

- $P \land Q$
- $P$
- Therefore, $Q$. 
Given two true statements, we can conclude using pure logic that a third is true. This process is called **logical inference**.

- \( P \implies Q \)
- \( P \)
- Therefore, \( Q \).

- \( P \implies Q \)
- \( \sim Q \)
- Therefore, \( \sim P \).

- \( P \lor Q \)
- \( \sim (P \land Q) \)
- \( P \land Q \)
- Therefore, \( Q \).
- Also therefore, \( P \).
Logical Inference

Definition

Given two true statements, we can conclude using pure logic that a third is true. This process is called **logical inference**.

- $P \Rightarrow Q$
- $P$
- Therefore, $Q$.
- $P \Rightarrow Q$
- $\sim Q$
- Therefore, $\sim P$.
- $P \lor Q$
- $\sim P$
- Therefore, $Q$.
- $P \land Q$
- Therefore, $Q$.
- Also therefore, $P$.
- $P$
Logical Inference

Definition
Given two true statements, we can conclude using pure logic that a third is true. This process is called **logical inference**.

- \( P \implies Q \)
- \( P \)
- Therefore, \( Q \).
- \( P \implies Q \)
- \( \sim Q \)
- Therefore, \( \sim P \).
- \( P \lor Q \)
- \( \sim P \)
- Therefore, \( Q \).
- \( P \land Q \)
- Therefore, \( Q \).
- Also therefore, \( P \).
- \( P \)
- Therefore, \( P \lor Q \).
Logical Inference

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Given two true statements, we can conclude using pure logic that a third is true. This process is called **logical inference**.

- \( P \implies Q \)
- \( P \)
- Therefore, \( Q \).

- \( P \implies Q \)
- \( \sim Q \)
- Therefore, \( \sim P \).

- \( P \lor Q \)
- \( \sim (P \land Q) \)
- \( P \)
- Therefore, \( P \lor Q \).

- \( P \land Q \)
- \( \sim P \)
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Logical Inference

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Given two true statements, we can conclude using pure logic that a third is true. This process is called **logical inference**.

- $P \implies Q$
- $P$
- Therefore, $Q$.

- $P \implies Q$
- $\sim Q$
- Therefore, $\sim P$.

- $P \lor Q$
- $\sim (P \land Q)$
- Therefore, $\sim Q$.

- $P \land Q$
- Therefore, $Q$.
- Also therefore, $P$.

- $P$
- Therefore, $P \lor Q$.
Transition to Proofs

We won’t usually write out these formal symbols while we’re proving something, but the logic behind them is always in the back of our minds.