Introduction

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> *In proving theorems, we apply logic to information that is considered obviously true (such as "Any two points determine exactly one line.") or is already known to be true (e.g., the Pythagorean theorem). If our logic is correct, then anything we deduce from such information will also be true (or at least as true as the "obviously true" information we began with).*

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If you are the prime minister, then you are human.

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Therefore, Justin Trudeau is human.

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I am not prime minister. Am I human?

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Example:
No mammal can fly.

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A pig is a mammal.

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I am not prime minister. Am I human?

Example:
No mammal can fly.
A pig is a mammal.
Therefore, a pig cannot fly. (True)
A bat is a mammal, therefore a bat cannot fly. (False)

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Definition

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$6 \times 3 = 18$

6×3

For every finite set A, $|\mathcal{P}(A)| = 2^{|A|}$.

For every real number x, $\sqrt{x^2} = x$.

Let x be a real number.

Multiply by the conjugate.

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We often give statements single-letter names, as we do for variables.
P: $6 \times 3 = 18$

We don't always know whether a statement is true or false.

Definition

Open sentence: truth depends on a variable.

Example: x is prime.

This is not a statement, at least not until we know what x is.

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Goldbach Conjecture

Every even integer greater than 2 is a sum of two prime numbers.

$4 = 2 + 2$, $6 = 3 + 3$, $8 = 3 + 5$, $20 = 7 + 13$

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The Goldbach Conjecture is a statement. (We don't know whether it is true or false, but we know it is one of them!)

And + Truth Tables

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Q	**R**	**Q ∧ R**
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T	F	
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True or False: 3 is prime or 5 is prime.
True or False: 3 is prime or 10 is prime.
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Or, Not

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T	T	T	T	F	F
T	F	T	F	T	T
F	T				
F	F				

Or, Not

P	**Q**	**P** $\vee$ **Q**
T	T	T
T	F	T
F	T	T
F	F	F

What if we want to express "P or Q, but not both, are true."
That is: $\underbrace{P \vee Q}_{\text{or}}$ is true, and $\underbrace{P \wedge Q}_{\text{and}}$ is false.

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T	T	T	T	F	F
T	F	T	F	T	T
F	T	T			
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T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F		
F	F				

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T	F	T
F	T	T
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T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	
F	F				

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F	T	T	F	T	T
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F	F	F	F	T	F

Conditional Statements ("if-then", implications)

Definition

For statements P and Q, the statement "If P then Q," written $P \Rightarrow Q$, is true *unless* P is true and Q is false.
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Q: n is divisible by 3.

$n = 27$

P	**Q**	**P ⇒ Q**	

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P	**Q**	**P ⇒ Q**	
T			

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P: n is divisible by 9
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$n = 27$
$n = 10$

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T	T	T	

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T	T	T	
F			

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T	T	T	T
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Conditional Example

P: $f(x)$ is a continuous function on $[a, b]$.
Q: N is a number strictly between $f(a)$ and $f(b)$.
R: There exists some number $c \in (a, b)$ such that $f(c) = N$.

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P: $f(x)$ is a continuous function on $[a, b]$.
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Intermediate Value Theorem

$$(P \wedge Q) \Rightarrow R$$

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$f(x) = \frac{1}{x^2}$
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P	**Q**	**R**	$(\mathbf{P} \wedge \mathbf{Q}) \Rightarrow \mathbf{R}$

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P	**Q**	**R**	$(\mathbf{P} \wedge \mathbf{Q}) \Rightarrow \mathbf{R}$
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F			T

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F	F	F	T
T			

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F	F	T	T
F	F	F	T
T	T		

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P	Q	R	$(P \wedge Q) \Rightarrow R$
F	F	T	T
F	F	F	T
T	T	T	

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If $A \Rightarrow B$ were false for some values of n, then it would not be a theorem.

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Further Examples with Conditional Statements

(the TV is made entirely of cheese)	(the TV does not turn on)
(you are in Canada)	(you are in Vancouver, BC)
$(x > 3)$	$(x \geq 3)$
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Biconditional Statements (if and only if)

Definition

A statement of the form $P \Leftrightarrow Q$ (meaning $P \Rightarrow Q$ AND $Q \Rightarrow P$) is **biconditional**. It means that P is true if and only if Q is true. In this case, we think of P and Q as being equivalent.

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Conditional or Biconditional?

(n is an even integer), (n is an integer divisible by 2)

(T is teaching our class right now), (T is named Elyse)

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Further Examples with (Bi-)Conditional Statements

$(x = 0)$	$(4x = 0)$
$(x^2 - 4 = 0)$	$(x = 2)$
$(x \text{ is an integer})$	$(x - 17 \text{ is an integer})$
$(x \text{ is odd})$	$(1 + x \text{ is even})$
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Suppose you have a statement that involves 5 separate statements, for example:

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Truth Tables

Suppose you have a statement that involves 5 separate statements, for example:

$$(A \wedge B \wedge C) \vee (\sim A \wedge D) \vee (\sim B \wedge E)$$

How many rows will be in the truth table?

A	B	C	D	E	$(A \wedge B \wedge C) \vee (\sim A \wedge D) \vee (\sim B \wedge E)$
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Logical Equivalence and Truth Tables

Definition

Two statements are logically equivalent if (and only if) they have exactly the same values in a truth table.

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F	T	T	F	F	T	T
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Logical Equivalence

Which of the following are logically equivalent statements?

- $P \Rightarrow Q$
- $\sim P \wedge \sim Q$
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P

$\boxed{P \Rightarrow Q}$

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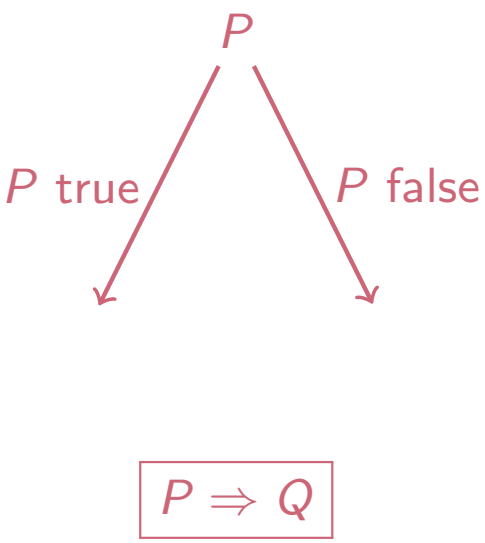

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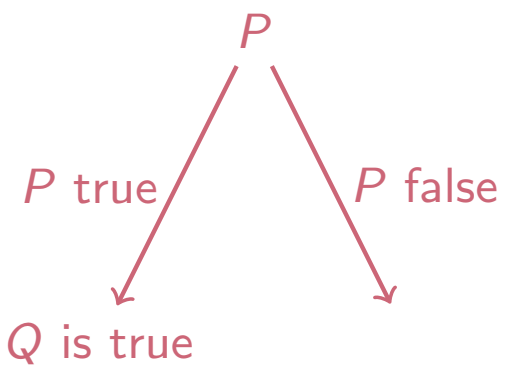

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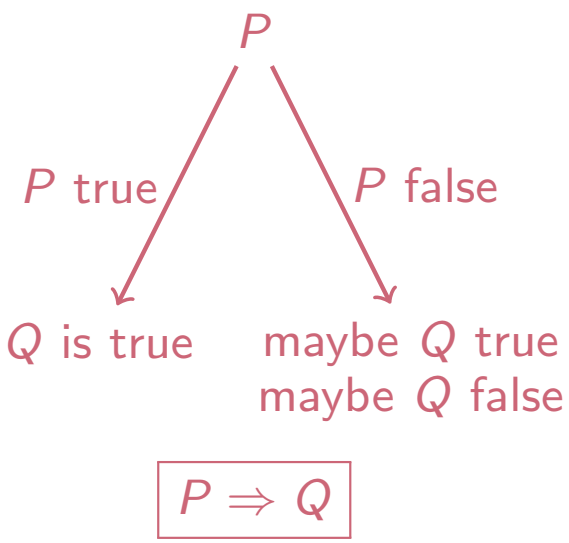

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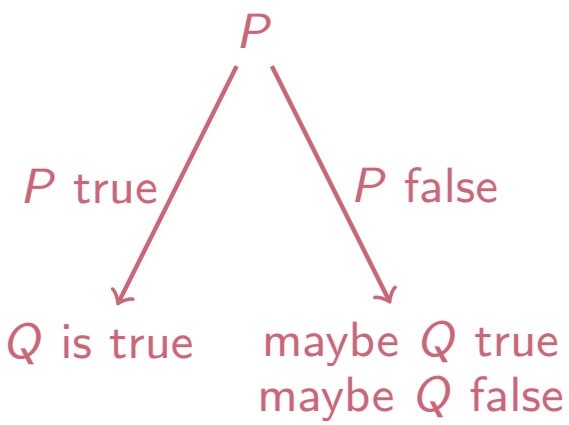

$$\boxed{P \Rightarrow Q}$$

$$\sim Q \Rightarrow \sim P$$

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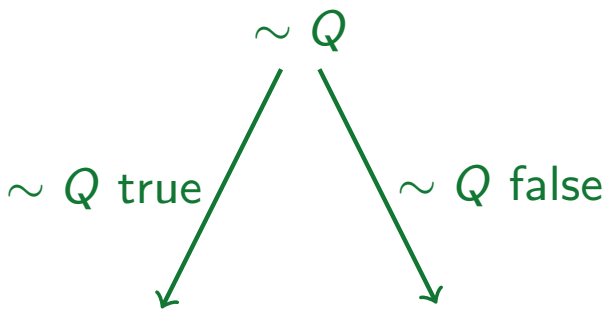

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$$\begin{tikzcd}
 & \sim Q \arrow[dl, "\sim Q \text{ true}"'] \arrow[dr, "\sim Q \text{ false}"] & \\
\sim P \text{ is true} & &
\end{tikzcd}$$

$$\boxed{\sim Q \Rightarrow \sim P}$$

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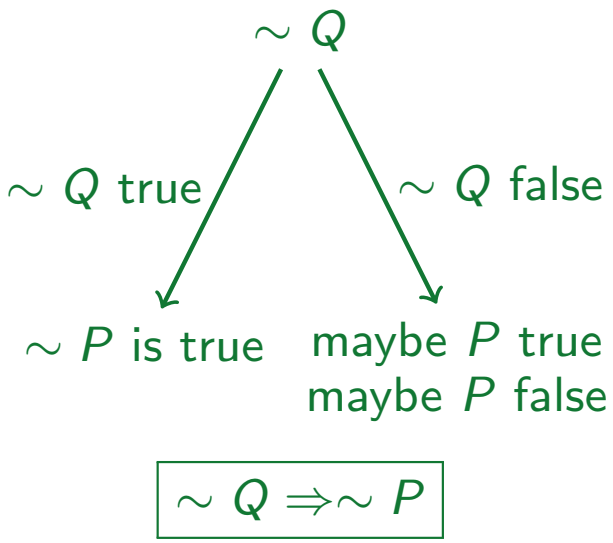

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Contrapositive:

- $(P \Rightarrow Q) \Leftrightarrow ((\sim Q) \Rightarrow (\sim P))$

$\sim Q$

$\sim Q$ true → $\sim P$ is true

$\sim Q$ false → maybe P true, maybe P false

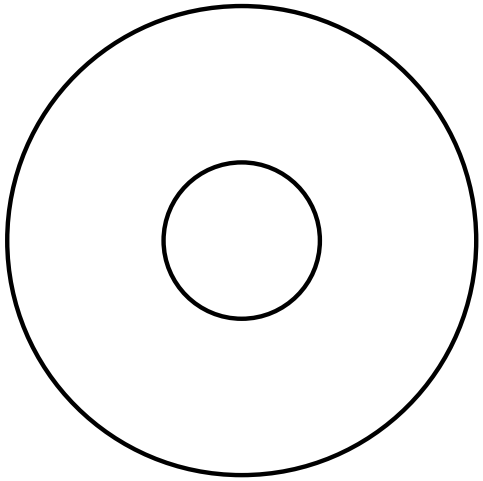
$$\boxed{\sim Q \Rightarrow \sim P}$$

$P \Rightarrow Q$

Implication (Conditional Statements) and the Contrapositive

$$(P \Rightarrow Q) \Leftrightarrow ((\sim Q) \Rightarrow (\sim P))$$

Let the Venn diagram below represent two sets: the set of all instances in which statement P is true, and the set of all instances in which Q is true. If $P \Rightarrow Q$, label P and Q below.



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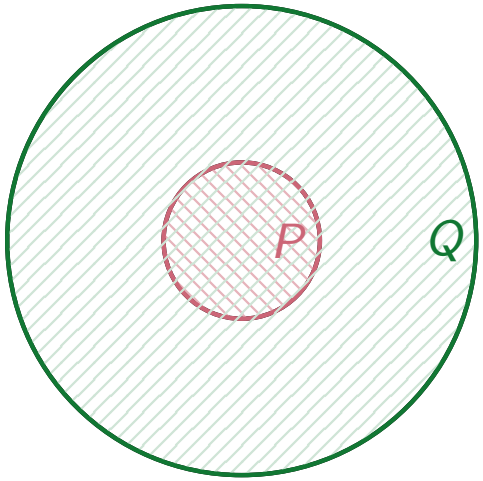

Common Equivalences

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  If you are prime minister, then you are a human.

Common Equivalences

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  If you are prime minister, then you are a human.
  If you are not a human, then you are not prime minister.

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  If you are prime minister, then you are a human.
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  If you are the murderer, then you drove a blue van yesterday.

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  If you are the murderer, then you drove a blue van yesterday.
  Jin did not drive a blue van yesterday. Can Jin be the murderer?

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Contrapositive:

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If you are prime minister, then you are a human.
If you are not a human, then you are not prime minister.

If you are the murderer, then you drove a blue van yesterday.
Jin did not drive a blue van yesterday. Can Jin be the murderer?
Logically, Jin is definitely not the murderer.

Common Equivalences

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Let $f(x)$ be a differentiable function. If $f(x)$ has at least two roots, then $f'(x) = 0$ somewhere. (Rolle's Theorem)

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Let $f(x)$ be a differentiable function. If $f(x)$ has at least two roots, then $f'(x) = 0$ somewhere. (Rolle's Theorem)
Let $f(x)$ be a differentiable function. If $f'(x)$ is never zero, then $f(x)$ has at most one root.
Example: $f(x) = e^x + x - 1$

A Statement is Logically Equivalent to its Contrapositive

For each statement below, choose the statement that is logically equivalent.

$$P \Rightarrow Q$$

(a) $Q \Rightarrow P$
(b) $\sim P \Rightarrow \sim Q$
(c) $\sim Q \Rightarrow \sim P$

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If it is a square, then it is a rectangle.

(a) If it is a rectangle, then it is a square.
(b) If is it not a square, then it is not a rectangle.
(c) If it is not a rectangle, then it is not a square.
(d) It is a square if and only if it is a rectangle.
(e) It is a square if and only if it is a square.

A Statement is Logically Equivalent to its Contrapositive

For each statement below, choose the statement that is logically equivalent.

$$P \Rightarrow Q$$

(a) $Q \Rightarrow P$
(b) $\sim P \Rightarrow \sim Q$
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If it is a square, then it is a rectangle.

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For each statement below, choose the statement that is logically equivalent.

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(a) If it is a rectangle, then it is a square.
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(d) It is a square if and only if it is a rectangle.
(e) It is a square if and only if it is a square.

If you cook, then you clean.

(a) If you do not cook, then you do not clean.
(b) If you do not clean, then you do not cook.
(c) You cook if and only if you clean.
(d) If you do not clean, you cooked.

A Statement is Logically Equivalent to its Contrapositive

For each statement below, choose the statement that is logically equivalent.

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(c) You cook if and only if you clean.
(d) If you do not clean, you cooked.

A Statement is Logically Equivalent to its Contrapositive

For each statement below, choose the statement that is logically equivalent.

If the year is 1999, then we are partying.

(a) If we are not partying, then the year is not 1999.
(b) If the year is not 1999, then we are not partying.
(c) We are partying if and only if the year is 1999.

If you aren't part of the solution, you're part of the problem.

(a) If you are a part of the problem, then you are not a part of the solution.
(b) You are a part of the solution if and only if you are not a part of the problem.
(c) If you are not a part of the problem, then you are a part of the solution.
(d) (none of the above)

If it's Tuesday, then it is raining.

(a) If it is not raining, then it is not Tuesday.
(b) If it is raining, then it is Tuesday.
(c) It is raining if and only if it is Tuesday.

A Statement is Logically Equivalent to its Contrapositive

For each statement below, choose the statement that is logically equivalent.

If the year is 1999, then we are partying.

(a) **If we are not partying, then the year is not 1999.**
(b) If the year is not 1999, then we are not partying.
(c) We are partying if and only if the year is 1999.

If you aren't part of the solution, you're part of the problem.

(a) If you are a part of the problem, then you are not a part of the solution.
(b) You are a part of the solution if and only if you are not a part of the problem.
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(d) (none of the above)

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- $\forall x \in \mathbb{N}, \forall y \in \mathbb{N}, x - y \in \mathbb{Z}$.
- $\forall x \in \mathbb{R} - \{0\}, \exists y \in \mathbb{R}$ such that $xy = 1$.
- $\exists x \in \mathbb{R} - \{0\}$ such that $\forall y \in \mathbb{R}, xy = 1$.
- $\forall x \in \mathbb{Q}, \forall n \in \mathbb{N}, \exists z \in \mathbb{Z}$ such that $x = \frac{z}{n}$.
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- $\exists x \in \mathbb{R} - \{0\}$ such that $\forall y \in \mathbb{R}, xy = 1$. False: $y = 0$
- $\forall x \in \mathbb{Q}, \forall n \in \mathbb{N}, \exists z \in \mathbb{Z}$ such that $x = \frac{z}{n}$. False: $x = \frac{1}{3}, n = 2$
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Fill in the Blank

Fill in each of the boxes with $\forall$, $\exists$, or $\nexists$. If multiple answers are possible, choose the best.

$\square$ integer n, $2n$ is even.

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$\square$ real $x \neq 0$, $\square$ real y such that $xy = 1$.

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$\boxed{\nexists}$ $x \in \mathbb{R}$ such that $\forall y \in \mathbb{R}$, $xy \neq 0$. No x has $xy \neq 0$ when $y = 0$.

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Translating English to Symbolic Logic

Every rational number is also a real number.

If x is a rational number, then x^2 is rational as well.

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Every integer is even or odd.

No real number is both even and irrational.

Every quadratic function with real coefficients has at least one (possibly complex) root.
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x is even, but y is odd

Whenever x is a perfect square, x^3 is as well.

P is true only if Q is true.
(For example: "I sing only if I'm in the shower.")

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$Q \Rightarrow P$
$P \Leftarrow Q$

Negating Statements using DeMorgan's Laws

DeMorgan's Laws

$$\sim (P \wedge Q) \quad \Leftrightarrow \quad (\sim P) \vee (\sim Q)$$

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Ted isn't smart or handsome.
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R: Everything in the library is also on the internet.

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S: There exists a human who has walked on the moon.
$\sim S$: No human has walked on the moon.
$\sim S$: For every human H, H has not walked on the moon.

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$\sim S$: For every human H, H has not walked on the moon.

$$\sim (\forall x \in A, P(x)) = \exists x \in A, \sim P(x)$$
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Negating Statements with Multiple Quantifiers

$$\sim (\forall x \in A, P(x)) = \exists x \in A, \sim P(x)$$
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P: $\forall x \in \mathbb{R}$, $\exists y \in \mathbb{R}$, $y^3 = x$.
That is: P says that every real number has a real cube root.

Q: $\exists x \in \mathbb{Z}$, $\forall y \in \mathbb{R}$, $xy = 0$.
That is: there is some integer x such that $xy = 0$ for every real number y.

$\forall A \subseteq \mathbb{N}$, $\exists a \in A$, $\forall b \in A$ $a \leq b$
Every subset of the natural numbers has a smallest element.

$\forall x \in \mathbb{R}, \forall y \in \mathbb{Z}, \exists z \in \{-5, 3.8\}, xyz \geq 0$

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$\exists A \subseteq \mathbb{N}, \forall a \in A, \exists b \in A, a > b$
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$\exists x \in \mathbb{R}, \exists y \in \mathbb{Z}, \forall z \in \{-5, 3.8\}, xyz < 0$
$\exists x \in \mathbb{R}, \exists y \in \mathbb{Z}, [(-5xy < 0) \wedge (3.8xy < 0)]$

Negating Conditional Statements

$\forall x \in \mathbb{R}, \left[\frac{x}{9} \in \mathbb{Z} \Rightarrow \frac{x}{3} \in \mathbb{Z}\right]$

$\forall x \in \mathbb{Z}, \left[x < a \Rightarrow x^2 < a^2\right]$

$\forall x, y, z, n \in \mathbb{N}, [(x^n + y^n = z^n) \Rightarrow n \in \{1, 2\}]$.
Fermat's Last Theorem

$\forall$ function $f(x)$, $\forall a < b \in \mathbb{R}$, $[f(x)$ continuous on $[a, b]] \Rightarrow$
$[\forall N$ between $f(a)$ and $f(b), \exists c \in (a, b), f(c) = N]$
Intermediate Value Theorem

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Logical Inference

Definition

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- $\sim P$
- Therefore, Q.

- $P \wedge Q$
- Therefore, Q.
- Also therefore, P.

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Given two true statements, we can conclude using pure logic that a third is true. This process is called **logical inference**.

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- P
- Therefore, Q.

- $P \Rightarrow Q$
- $\sim Q$
- Therefore, $\sim P$.

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- P

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- Therefore, Q.

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- Therefore, Q.
- Also therefore, P.

- P
- Therefore, $P \vee Q$.

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Given two true statements, we can conclude using pure logic that a third is true. This process is called **logical inference**.

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- P
- Therefore, Q.

- $P \Rightarrow Q$
- $\sim Q$
- Therefore, $\sim P$.

- $P \vee Q$
- $\sim P$
- Therefore, Q.

- $P \wedge Q$
- Therefore, Q.
- Also therefore, P.

- P
- Therefore, $P \vee Q$.

- $\sim (P \wedge Q)$
- P

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- Therefore, $\sim P$.

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- Therefore, Q.

- $P \wedge Q$
- Therefore, Q.
- Also therefore, P.

- P
- Therefore, $P \vee Q$.

- $\sim (P \wedge Q)$
- P
- Therefore, $\sim Q$.

Transition to Proofs

We won't usually write out these formal symbols while we're proving something, but the logic behind them is always in the back of our minds.