Week 5: Circuits

Course Notes: 3.5

Goals: Use linear algebra to determine voltage drops and branch currents.

Components in Resistor Networks

- 9V voltage source
- 3A current source (inductor at an instant)
- 6Ω resistor

\[ V = IR \]
Setup: Given: Resistance of resistors; voltage across voltage sources; current through current sources.
Find: currents through each resistor and each voltage source; voltage drops across each current source.

Kirchhoff's Laws

1. The sum of voltage drops around any closed loops in the network must be zero.
2. For any node, current in equals current out.

Notes
\[ i_1 \approx 0.2449, \quad i_2 \approx 0.1114, \quad i_3 \approx 0.1166 \]

Equations from previous slide:

\[ i_1 \text{ loop: } -10 + i_1 + 25(i_1 - i_2) + 50(i_1 - i_3) = 0 \]
\[ i_2 \text{ loop: } 25(i_2 - i_1) + 30i_2 + (i_2 - i_3) = 0 \]
\[ i_3 \text{ loop: } 50(i_3 - i_1) + (i_3 - i_2) + 55i_3 = 0 \]

\[
\begin{align*}
76i_3 - 25i_2 - 50i_3 &= 10 \\
-25i_1 + 56i_2 - i_3 &= 0 \\
-50i_1 - i_2 + 106i_3 &= 0
\end{align*}
\]
Equations from Previous Slide:

\( i_1 \) loop: \( 10 + 2(i_1 - i_4) + (i_1 - i_2) = 0 \)
\( i_2 \) loop: \( 2i_2 + (i_2 - i_1) + 4(i_2 - i_4) = 0 \)
\( i_3 \) loop: \( 10 + 4(i_3 - i_2) + 3(i_3 - i_4) = 0 \)
\( i_4 \) loop: \( 5i_4 + 3(i_4 - i_3) + 2(i_4 - i_2) = 0 \)

\[
\begin{align*}
3i_1 &- i_2 + 0i_3 - 2i_4 = -10 \\
-h_1 + 7i_2 - 4i_3 + 0i_4 & = 0 \\
0i_1 - 4i_2 + 7i_3 - 3i_4 & = -10 \\
-2i_1 + 0i_2 - 3i_3 + 10i_4 & = 0
\end{align*}
\]
Let $E$ be the voltage drop across the current source.

Equations from previous slide:

**Current Source:** $5 = \dot{i} - \dot{i}_2$

$\dot{i}_1$ Loop: $-10 + 3(\dot{i} - \dot{i}_1) + 2(\dot{i}_1 - \dot{i}_2) = 0$

$\dot{i}_2$ Loop: $2(\dot{i}_2 - \dot{i}_1) + E = 0$

$\dot{i}_3$ Loop: $-E + 3(\dot{i} - \dot{i}_3) + \dot{i}_3 = 0$

\[
\begin{align*}
0\dot{i}_1 - \dot{i}_2 + \dot{i}_3 + 0E &= 5 \\
5\dot{i}_1 - 2\dot{i}_2 - 3\dot{i}_3 + 0E &= 10 \\
-2\dot{i}_1 + 2\dot{i}_2 + 0\dot{i}_3 + E &= 0 \\
-3\dot{i}_1 + 0\dot{i}_2 + 4\dot{i}_3 - E &= 0
\end{align*}
\]

\[
\begin{align*}
\dot{i}_1 &\approx -8.8571, & \dot{i}_2 &\approx 4.1429, & \dot{i}_3 &\approx -3.8571, \\
E_1 &\approx 52.5714, & E_2 &\approx 42.5714
\end{align*}
\]
Equations from previous slide:

5A Current Source: \( i_3 - i_1 = 5 \)

8A Current Source: \( i_2 - i_3 = 8 \)

\( i_1 \) Loop: \( 3i_1 + 2(i_2 - i_3) + E_1 = 0 \)

\( i_2 \) Loop: \( 2(i_2 - i_1) + 4i_2 - E_2 = 0 \)

\( i_3 \) Loop: \( -E_1 + E_2 + 10 = 0 \)

\[ -i_1 + 0i_2 + i_3 + 0E_1 + 0E_2 = 5 \]
\[ 0i_1 + i_2 - i_3 + 0E_1 + 0E_2 = 8 \]
\[ 5i_1 - 2i_2 + 0i_3 + E_1 + 0E_2 = 0 \]
\[ -2i_1 + 6i_2 + 0i_3 + 0E_1 - E_2 = 0 \]
\[ 0i_1 + 0i_2 + 0i_3 - E_1 + E_2 = -10 \]
clockwise: $i_1 = -7.5$, $i_2 = -0.625$, $i_3 = 0$, $i_4 = 0.625$, $i_5 = 7.5$
What voltage should the voltage source have, in order for there to be no current across it?

What resistance should the top resistor have, if you want each wire touching the centre to have current 5A?

Replace ONE unmarked resistor (with a different resistor or a different component) so that the current through the marked resistor is zero.
Find all ways to change the resistances of the non-marked resistors so that the current flowing through the marked resistor is zero.
Justify your answer with algebra.