Course Notes: 2.1-2.3
Course Outline: Week 1
What is a Vector?

Vectors are used to describe quantities with a magnitude (length) and a direction.
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\[ F_1 = F_2 = G \frac{m_1 \times m_2}{r^2} \]

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Notice: a vector doesn’t intrinsically have a position, although we can assign it one in context.
Scalar Multiplication

Multiplying a vector \( \mathbf{a} \) by a scalar \( s \) results in a vector with length \( |s| \) times the length of \( \mathbf{a} \). The new vector \( s\mathbf{a} \) points in the same direction if \( s \) is positive, and in the opposite direction if \( s \) is negative.

If the length of \( \mathbf{a} \) is 1 unit, then the length of \( 2\mathbf{a} \) is 2. What is the length of \( -1\mathbf{a} \): is it 1, or -1?
Vector Operations: Multiply by a Number

Scalar Multiplication

Multiplying a vector $\mathbf{a}$ by a scalar $s$ results in a vector with length $|s|$ times the length of $\mathbf{a}$. The new vector $s\mathbf{a}$ points in the same direction if $s$ is positive, and in the opposite direction if $s$ is negative.
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If the length of $\mathbf{a}$ is 1 unit, then the length of $2\mathbf{a}$ is 2. What is the length of $-1\mathbf{a}$: is it 1, or -1?
Vector Addition

To add vectors \( \mathbf{a} \) and \( \mathbf{b} \), we can slide the tail of \( \mathbf{a} \) to sit at the head of \( \mathbf{b} \), and take \( \mathbf{a} + \mathbf{b} \) to be the vector with tail where the tail of \( \mathbf{b} \) is, and head where the head of \( \mathbf{a} \) is. This is equivalent to making a parallelogram out of \( \mathbf{a} \) and \( \mathbf{b} \) (with the same tail) and taking the diagonal (again with the same tail) to be the vector \( \mathbf{a} + \mathbf{b} \).
Vector Operations: Adding Vectors

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Vector Addition

In each case, sketch a vector $\mathbf{b}$ such that $\mathbf{a} + \mathbf{b} = \mathbf{c}$. 
Vector Addition

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\begin{itemize}
  \item \begin{figure}
      \centering
      \includegraphics[width=0.3\textwidth]{vector_addition_1}
  \end{figure}
  \item \begin{figure}
      \centering
      \includegraphics[width=0.3\textwidth]{vector_addition_2}
  \end{figure}
  \item \begin{figure}
      \centering
      \includegraphics[width=0.3\textwidth]{vector_addition_3}
  \end{figure}
  \item \begin{figure}
      \centering
      \includegraphics[width=0.3\textwidth]{vector_addition_4}
  \end{figure}
\end{itemize}
In each case, sketch a vector \( \mathbf{b} \) such that \( \mathbf{a} + \mathbf{b} = \mathbf{c} \).
In each case, sketch a vector $\mathbf{b}$ such that $\mathbf{a} + \mathbf{b} = \mathbf{c}$. 

![Vector Addition Diagram]
In each case, sketch a vector $\mathbf{b}$ such that $\mathbf{a} + \mathbf{b} = \mathbf{c}$. 

![Vector Addition Diagram]
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Vector Addition

In each case, sketch a vector $b$ such that $a + b = c$. 
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Vector Addition

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Example

Suppose we add a vector $\mathbf{a}$ to the vector $-3\mathbf{a}$. What should be the resulting vector?
Vector Operations

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As we might expect, $\mathbf{a} - 3\mathbf{a} = -2\mathbf{a}$. 
Vector Operations

Example

Suppose we add a vector \( \mathbf{a} \) to the vector \( -3\mathbf{a} \). What should be the resulting vector?

\[
-3\mathbf{a} + \mathbf{a} = -2\mathbf{a}
\]
**Example**

Suppose we add a vector $\mathbf{a}$ to the vector $-3\mathbf{a}$. What should be the resulting vector?

As we might expect, $\mathbf{a} - 3\mathbf{a} = -2\mathbf{a}$. 
Limits of Sketching

Suppose a ship is sailing in the ocean. The current is pushing the ship at 5 knots per hour due east, while the wind is pushing this ship 3 knots per hour northwest. Rowers onboard are providing a force equal to 2 knots per hour east-southeast. What direction is the ship moving, and how fast?
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Time for coordinates.

See also: https://en.wikipedia.org/wiki/Wind_triangle
Coordinates and Vectors

\[ \begin{bmatrix} 3 \\ 2 \end{bmatrix} \]

\[ 3\mathbf{i} + 2\mathbf{j} = a \mathbf{i} \text{ and } \mathbf{j} \]

are unit vectors, and we can write any vector in \( \mathbb{R}^2 \) as a linear combination of them.

unit vector: length one

linear combination: any combination using only addition and scalar multiplication
Coordinates and Vectors

\[ \mathbf{a} = 3\mathbf{i} + 2\mathbf{j} \]

\(\mathbf{i}\) and \(\mathbf{j}\) are unit vectors, and we can write any vector in \(\mathbb{R}^2\) as a linear combination of them.

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Coordinates and Vectors

\[ \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \end{pmatrix} \]

\[ 3i + 2j = a \]

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**unit vector**: length one

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Coordinates and Vectors

\[ \begin{pmatrix} 3 \\ 2 \end{pmatrix} \]

\( \mathbf{i} \) and \( \mathbf{j} \) are unit vectors, and we can write any vector in \( \mathbb{R}^2 \) as a linear combination of them.

Unit vector: length one

Linear combination: any combination using only addition and scalar multiplication
Coordinates and Vectors

A vector can be represented as a linear combination of unit vectors $i$ and $j$. For example, the vector $[3, 2]$ can be expressed as $3i + 2j$. These unit vectors have length one, and any vector in $\mathbb{R}^2$ can be written as a linear combination of them.
Coordinates and Vectors

\[ [3, 2] \]

\[ \mathbf{a} \]

\[ \mathbf{i} \] and \[ \mathbf{j} \] are unit vectors, and we can write any vector in \( \mathbb{R}^2 \) as a linear combination of them.

A unit vector has length one.

A linear combination is any combination using only addition and scalar multiplication.
Coordinates and Vectors

\[ \begin{bmatrix} 3 \\ 2 \end{bmatrix} \]

\[ 3 \hat{i} + 2 \hat{j} = \mathbf{a} \]

\( \hat{i} \) and \( \hat{j} \) are unit vectors, and we can write any vector in \( \mathbb{R}^2 \) as a linear combination of them.

unit vector: length one

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Coordinates and Vectors

\[ \begin{pmatrix} 3 \\ 2 \end{pmatrix} \text{ and } \begin{pmatrix} 3 \\ 2 \end{pmatrix} \text{ are unit vectors, and we can write any vector in } \mathbb{R}^2 \text{ as a linear combination of them.} \]
Coordinates and Vectors

\[ \begin{align*}
3i + 2j &= a \\
i \text{ and } j \text{ are unit vectors, and we can write any vector in } \mathbb{R}^2 \text{ as a linear combination of them.}
\end{align*} \]
Coordinates and Vectors

\[ 3\mathbf{i} + 2\mathbf{j} = \mathbf{a} \]
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\(\mathbf{i}\) and \(\mathbf{j}\) are *unit vectors*, and we can write any vector in \(\mathbb{R}^2\) as a *linear combination* of them.
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*unit vector*: length one  
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Vector Operations on Coordinates
Vector Operations on Coordinates

[Diagram showing vectors a and b on a coordinate plane.]
Vector Operations on Coordinates
Vector Operations on Coordinates

\[
\begin{bmatrix}
3 \\
1
\end{bmatrix} + \begin{bmatrix}
1 \\
2
\end{bmatrix} = \begin{bmatrix}
4 \\
3
\end{bmatrix}
\]
Vector Operations on Coordinates

\[
\begin{pmatrix}
3 \\
1 \\
7
\end{pmatrix} + 
\begin{pmatrix}
1 \\
2 \\
0
\end{pmatrix} = 
\begin{pmatrix}
4 \\
3 \\
7
\end{pmatrix}
\]
Vector Operations on Coordinates

\[
\begin{bmatrix}
3 \\
1 \\
7 \\
10
\end{bmatrix}
+ \begin{bmatrix}
1 \\
2 \\
0 \\
20
\end{bmatrix}
= \begin{bmatrix}
4 \\
3 \\
7 \\
30
\end{bmatrix}
\]
Vector Operations on Coordinates
Vector Operations on Coordinates
Vector Operations on Coordinates

\[
2 \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}
\]
Vector Operations on Coordinates

\[ \begin{bmatrix} 3 \\ 1 \\ 6 \\ 9 \end{bmatrix} \times \frac{1}{3} = \begin{bmatrix} 1 \\ 1/3 \\ 2 \\ 3 \end{bmatrix} \]
Properties of Vector Addition and Scalar Multiplication

(Notes: 2.2.3)
Let \( \mathbf{0} \) be the zero vector: this is the vector whose components are all zero. Let \( \mathbf{a}, \mathbf{b}, \) and \( \mathbf{c} \) be vectors, and let \( s \) and \( t \) be scalars. The following facts about vector addition, and multiplication of vectors by scalars, are true:

1. \( \mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a} \)
2. \( \mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c} \)
3. \( \mathbf{a} + \mathbf{0} = \mathbf{a} \)
4. \( \mathbf{a} + (\mathbf{-a}) = \mathbf{0} \)
5. \( s(\mathbf{a} + \mathbf{b}) = sa + sb \)
6. \( (s + t)a = sa + ta \)
7. \( (st)a = s(ta) \)
8. \( 1a = a \)
Vectors versus Coordinates

Because we write vectors like coordinates, we will often use them interchangeably with points. You will have to figure this out from context.
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Example: Let \( \mathbf{a} \) be a fixed, nonzero vector. Describe and sketch the sets of points in two dimensions:

\[ \{ s \mathbf{a} : s \in \mathbb{R} \} \]
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\{ sa : s \in \mathbb{R} \}
\]
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Example: Let $\mathbf{a}$ be a fixed, nonzero vector. Describe and sketch the sets of points in two dimensions:

$$\{s\mathbf{a} : s \in \mathbb{R}\}$$

Example: Let $\mathbf{a}$ and $\mathbf{b}$ be fixed, nonzero vectors. Describe and sketch the sets of points in two dimensions:

$$\{s\mathbf{a} + t\mathbf{b} : s, t \in \mathbb{R}\}$$
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\[
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\]

See

http://thejuniverse.org/PUBLIC/LinearAlgebra/LOLA/spans/two.html
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Example: Let $\mathbf{a}$ and $\mathbf{b}$ be fixed, nonzero vectors. Describe and sketch the sets of points in three dimensions:

$$\{s\mathbf{a} + t\mathbf{b} : s, t \in \mathbb{R}\}$$
In each case below, show that the vector $\mathbf{c}$ can be written as $s\mathbf{a} + t\mathbf{b}$ for some $s, t \in \mathbb{R}$. 
In each case below, show that the vector $\mathbf{c}$ can be written as $s\mathbf{a} + t\mathbf{b}$ for some $s, t \in \mathbb{R}$. 
In each case below, show that the vector \( \mathbf{c} \) can be written as \( s\mathbf{a} + t\mathbf{b} \) for some \( s, t \in \mathbb{R} \).
Vectors versus Coordinates

In each case below, show that the vector $\mathbf{c}$ can be written as $s\mathbf{a} + t\mathbf{b}$ for some $s, t \in \mathbb{R}$. 

![Diagram showing vectors a, b, and c connected by lines to demonstrate the vector equation.](image-url)
Vectors versus Coordinates

In each case below, show that the vector $\mathbf{c}$ can be written as $s\mathbf{a} + t\mathbf{b}$ for some $s, t \in \mathbb{R}$. 
Vectors versus Coordinates

In each case below, show that the vector $c$ can be written as $sa + tb$ for some $s, t \in \mathbb{R}$.
In each case below, show that the vector \( \mathbf{c} \) can be written as \( s \mathbf{a} + t \mathbf{b} \) for some \( s, t \in \mathbb{R} \).
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Vectors versus Coordinates

In each case below, show that the vector \( \mathbf{c} \) can be written as \( s \mathbf{a} + t \mathbf{b} \) for some \( s, t \in \mathbb{R} \).
In each case below, show that the vector $\mathbf{c}$ can be written as $s\mathbf{a} + t\mathbf{b}$ for some $s, t \in \mathbb{R}$.
In each case below, show that the vector $\mathbf{c}$ can be written as $s\mathbf{a} + t\mathbf{b}$ for some $s, t \in \mathbb{R}$.
Let \( \mathbf{a} \) and \( \mathbf{b} \) be fixed, nonzero vectors.

- Give an expression for the midpoint of the line segment halfway between \( \mathbf{a} \) and \( \mathbf{b} \).
Let \( \mathbf{a} \) and \( \mathbf{b} \) be fixed, nonzero vectors.

- Give an expression for the midpoint of the line segment halfway between \( \mathbf{a} \) and \( \mathbf{b} \).
- Give an expression for the point that is one-third of the way along the line segment between \( \mathbf{a} \) and \( \mathbf{b} \).
- What is the geometric interpretation of the following set of points:
  \[ \{s\mathbf{a} + (1 - s)\mathbf{b} : 0 \leq s \leq 1\} \]
- What is the geometric interpretation of the following set of points:
  \[ \{(1 - s)\mathbf{a} + s\mathbf{b} : 0 \leq s \leq 1\} \]
Geometric Aspects of Vectors

How long is the vector \[ \begin{bmatrix} 12 \\ 5 \end{bmatrix} \]?

The length of \[ \begin{bmatrix} 12 \\ 5 \end{bmatrix} \] is denoted \( \left\| \begin{bmatrix} 12 \\ 5 \end{bmatrix} \right\| \), and calculated as

\[
\left\| \begin{bmatrix} 12 \\ 5 \end{bmatrix} \right\| = \sqrt{12^2 + 5^2} = 13.
\]
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We also call this quantity the norm of the vector.

What about vectors with three coordinates?
How long is the vector \([ \begin{pmatrix} 12 \\ 5 \end{pmatrix} ]\)?

The length of \([ \begin{pmatrix} 12 \\ 5 \end{pmatrix} ]\) is denoted \(\| \begin{pmatrix} 12 \\ 5 \end{pmatrix} \|\), and calculated as:

\[\| \begin{pmatrix} 12 \\ 5 \end{pmatrix} \| = \sqrt{12^2 + 5^2} = \sqrt{169} = 13.\]

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What about vectors with three coordinates?
Geometric Aspects of Vectors

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How long is the vector \( \begin{bmatrix} 12 \\ 5 \end{bmatrix} \)?

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\[
\| \begin{bmatrix} 12 \\ 5 \end{bmatrix} \| = \sqrt{12^2 + 5^2} = 13.
\]

We also call this quantity the norm of the vector.

What about vectors with three coordinates?
\[
\mathbf{v} = \begin{bmatrix}
    a \\
    b \\
    c
\end{bmatrix}
\]

The magnitude of \( \mathbf{v} \) is:

\[
\| \mathbf{v} \| = \sqrt{a^2 + b^2 + c^2}
\]
\[ \mathbf{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \]

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\[ \mathbf{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \]

\[ \| \mathbf{v} \| = \sqrt{a^2 + b^2 + c^2} \]
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\[ \| \mathbf{v} \| = \sqrt{a^2 + b^2 + c^2} \]
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\[ \| \mathbf{v} \| = \sqrt{a^2 + b^2 + c^2} \]
\[ \mathbf{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \]

\[ \| \mathbf{v} \| = \sqrt{(a^2 + b^2 + c^2)} = \]
\[ \mathbf{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \]

\[ \|\mathbf{v}\| = \sqrt{\left(\sqrt{a^2 + b^2}\right)^2 + c^2} = \]
\[ \| \mathbf{v} \| = \sqrt{\left( \sqrt{a^2 + b^2} \right)^2 + c^2} = \sqrt{a^2 + b^2 + c^2} \]
Geometric Aspects of Vectors

How long is the vector \( \begin{pmatrix} 12 \\ 5 \end{pmatrix} \)?

The length of \( \begin{pmatrix} 12 \\ 5 \end{pmatrix} \) is denoted \( \| \begin{pmatrix} 12 \\ 5 \end{pmatrix} \| \), and calculated

\[ \| \begin{pmatrix} 12 \\ 5 \end{pmatrix} \| = \sqrt{12^2 + 5^2} = 13. \]

We also call this quantity the norm of the vector.

The length of \( \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \) is denoted \( \| \mathbf{a} \| \), and calculated

\[ \| \mathbf{a} \| = \sqrt{a_1^2 + a_2^2 + a_3^2} \]
Quick Concept Test

Let \( \mathbf{a} \) be a vector, and let \( s \) be a scalar. For each of the following expressions, decide whether it is a vector or a scalar.

A. \( \|\mathbf{a}\| \)
B. \( s\mathbf{a} \)
C. \( s\|\mathbf{a}\| \)
D. \( \|s\mathbf{a}\| \)
E. \( s + \mathbf{a} \)
F. \( s + \|\mathbf{a}\| \)
A **unit vector** is a vector of length one.

What is the unit vector in the direction of the vector $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$?
A **unit vector** is a vector of length one.

What is the unit vector in the direction of the vector \[
\begin{bmatrix} 3 \\ 4 \end{bmatrix}
\]?

If a vector is in the same direction as \[
\begin{bmatrix} 3 \\ 4 \end{bmatrix}
\], then it’s a (positive) scalar multiple of that vector. So, we want to divide by the length of our vector.

We compute the length of the vector: 
\[
\left\| \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\| = \sqrt{3^2 + 4^2} = 5.
\]

Then the vector \[
\frac{1}{5} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} \\ \frac{4}{5} \end{bmatrix}
\] is the unit vector in the same direction as the given vector.
## Dot Product

Given vectors \( \mathbf{a} = [a_1, \ldots, a_k] \) and \( \mathbf{b} = [b_1, \ldots, b_k] \), we define the dot product \( \mathbf{a} \cdot \mathbf{b} := a_1 b_1 + \cdots + a_k b_k \). Note \( \mathbf{a} \cdot \mathbf{b} \) is a number, not a vector.
Dot Product

Given vectors \( \mathbf{a} = [a_1, \ldots, a_k] \) and \( \mathbf{b} = [b_1, \ldots, b_k] \), we define the dot product \( \mathbf{a} \cdot \mathbf{b} := a_1 b_1 + \cdots + a_k b_k \). Note \( \mathbf{a} \cdot \mathbf{b} \) is a number, not a vector.

Example:
\[
\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} = -4 + 0 + 15 = 11
\]
## Dot Product

Given vectors \( \mathbf{a} = [a_1, \ldots, a_k] \) and \( \mathbf{b} = [b_1, \ldots, b_k] \), we define the dot product \( \mathbf{a} \cdot \mathbf{b} := a_1 b_1 + \cdots + a_k b_k \). Note \( \mathbf{a} \cdot \mathbf{b} \) is a number, not a vector.

**Example:**

\[
\begin{bmatrix}
2 \\
1 \\
5
\end{bmatrix}
\cdot
\begin{bmatrix}
-2 \\
0 \\
3
\end{bmatrix}
= -4 + 0 + 15 = 11
\]

**Note:** \( \mathbf{a} \cdot \mathbf{a} = \| \mathbf{a} \|^2 \).
Dot Product

Given vectors \( \mathbf{a} = [a_1, \ldots, a_k] \) and \( \mathbf{b} = [b_1, \ldots, b_k] \), we define the dot product \( \mathbf{a} \cdot \mathbf{b} := a_1 b_1 + \cdots + a_k b_k \). Note \( \mathbf{a} \cdot \mathbf{b} \) is a number, not a vector.

**Example:**
\[
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\]

**Note:** \( \mathbf{a} \cdot \mathbf{a} = \| \mathbf{a} \|^2 \).

\[
\begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} = 2^2 + 1^2 + 5^2
\]

\[
\| \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} \| = \sqrt{2^2 + 1^2 + 5^2}
\]
Properties of the Dot Product

Notes: p. 20
For nonzero vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$, zero vector $\mathbf{0}$, and scalar $s$:

1. $\mathbf{a} \cdot \mathbf{a} = ||\mathbf{a}||^2$
Properties of the Dot Product

Notes: p. 20

For nonzero vectors \( \mathbf{a} \), \( \mathbf{b} \), and \( \mathbf{c} \), zero vector \( \mathbf{0} \), and scalar \( s \):

1. \( \mathbf{a} \cdot \mathbf{a} = ||\mathbf{a}||^2 \)
2. \( \mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a} \)
3. \( \mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c} \)
4. \( s(\mathbf{a} \cdot \mathbf{b}) = (sa) \cdot \mathbf{b} \)
5. \( \mathbf{0} \cdot \mathbf{a} = 0 \)
Properties of the Dot Product

Notes: p. 20
For nonzero vectors \( \mathbf{a} \), \( \mathbf{b} \), and \( \mathbf{c} \), zero vector \( \mathbf{0} \), and scalar \( s \):

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3. \( \mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c} \)
4. \( s(\mathbf{a} \cdot \mathbf{b}) = (sa) \cdot \mathbf{b} \)
5. \( \mathbf{0} \cdot \mathbf{a} = 0 \)

6. \( \mathbf{a} \cdot \mathbf{b} = \| \mathbf{a} \| \| \mathbf{b} \| \cos \theta \), where \( \theta \) is the angle between \( \mathbf{a} \) and \( \mathbf{b} \)
7. \( \mathbf{a} \cdot \mathbf{b} = 0 \) if and only if \( \mathbf{a} = \mathbf{0} \), \( \mathbf{b} = \mathbf{0} \), or \( \mathbf{a} \) and \( \mathbf{b} \) are perpendicular
Properties of the Dot Product

Notes: p. 20
For nonzero vectors \( \mathbf{a} \), \( \mathbf{b} \), and \( \mathbf{c} \), zero vector \( \mathbf{0} \), and scalar \( s \):

1. \( \mathbf{a} \cdot \mathbf{a} = \| \mathbf{a} \|^2 \)
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Example: are \( \mathbf{a} \) and \( \mathbf{b} \) perpendicular?

\[
\begin{align*}
\bullet \quad \mathbf{a} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
\bullet \quad \mathbf{a} &= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}
\end{align*}
\]
Properties of the Dot Product

\( \mathbf{a} \cdot \mathbf{b} = 0 \) if and only if \( \mathbf{a} = \mathbf{0} \), \( \mathbf{b} = \mathbf{0} \), or \( \mathbf{a} \) and \( \mathbf{b} \) are perpendicular.

Example: are \( \mathbf{a} \) and \( \mathbf{b} \) perpendicular?

\[
\mathbf{a} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\]

No

\[
\mathbf{a} = [2, -1], \quad \mathbf{b} = [-3, 6]
\]

No

\[
\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}
\]

No

\[
\mathbf{a} = \begin{bmatrix} 2 \\ 1 \\ -2 \\ -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ -2 \\ 1 \\ 2 \end{bmatrix}
\]

Yes
Properties of the Dot Product

\( \mathbf{a} \cdot \mathbf{b} = 0 \) if and only if \( \mathbf{a} = \mathbf{0} \), \( \mathbf{b} = \mathbf{0} \), or \( \mathbf{a} \) and \( \mathbf{b} \) are perpendicular.

Example: are \( \mathbf{a} \) and \( \mathbf{b} \) perpendicular?

\[ \mathbf{a} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \]

No

\[ \mathbf{a} = [2, -1], \quad \mathbf{b} = [-3, 6] \]

No

\[ \mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} \]

No

\[ \mathbf{a} = \begin{bmatrix} 2 \\ 1 \\ -2 \\ -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ -2 \\ 1 \\ 2 \end{bmatrix} \]

Yes
\[ \mathbf{a} \cdot \mathbf{b} = \| \mathbf{a} \| \| \mathbf{b} \| \cos \theta \]
\[ \mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos \theta \]

Claim 1:
\[ \|\mathbf{a} - \mathbf{b}\|^2 = \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 - 2 \mathbf{a} \cdot \mathbf{b} \]

Claim 2:
\[ \|\mathbf{a} - \mathbf{b}\|^2 = \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 - 2 \|\mathbf{a}\| \|\mathbf{b}\| \cos \theta \]
\[ \mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\| \cos \theta \]

Claim 1:
\[ \|\mathbf{a} - \mathbf{b}\|^2 = \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 - 2\mathbf{a} \cdot \mathbf{b} \]

Claim 2:
\[ \|\mathbf{a} - \mathbf{b}\|^2 = \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 - 2\|\mathbf{a}\|\|\mathbf{b}\| \cos \theta \]

Then:
\[ \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 - 2\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 - 2\|\mathbf{a}\|\|\mathbf{b}\| \cos \theta \]
\[ -2\mathbf{a} \cdot \mathbf{b} = -2\|\mathbf{a}\|\|\mathbf{b}\| \cos \theta \]
\[ \mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\| \cos \theta \]
Claim 1:

\[ \|a - b\|^2 = \|a\|^2 + \|b\|^2 - 2a \cdot b \]

Proof:

\[ \|a - b\|^2 = (a - b) \cdot (a - b) \]
\[ = a^2 + b^2 - 2a \cdot b \]
\[ = \|a\|^2 + \|b\|^2 - 2a \cdot b \]
Claim 2:

$$\|a - b\|^2 = \|a\|^2 + \|b\|^2 - 2\|a\|\|b\| \cos \theta$$

Law of Cosines
Recall $\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\| \cos \theta$, where $\theta$ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

What is the angle between vectors $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 5 \end{bmatrix}$?
Recall $\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos \theta$, where $\theta$ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

What is the angle between vectors $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 5 \end{bmatrix}$?

\[
\begin{align*}
\begin{bmatrix} 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 5 \end{bmatrix} &= \|\begin{bmatrix} 2 \\ 1 \end{bmatrix}\| \|\begin{bmatrix} 5 \\ 5 \end{bmatrix}\| \cos \theta \\
10 + 5 &= \sqrt{2^2 + 1^2} \sqrt{5^2 + 5^2} \cos \theta \\
15 &= 5\sqrt{10} \cos \theta \\
\cos \theta &= \frac{3}{\sqrt{10}} \\
\theta &= \arccos \left( \frac{3}{\sqrt{10}} \right)
\end{align*}
\]
Projections

We apply a force to an object in the direction of $\mathbf{a}$, but we’re only concerned with the object’s movement in the direction of vector $\mathbf{b}$.
Projections

We apply a force to an object in the direction of $\mathbf{a}$, but we’re only concerned with the object’s movement in the direction of vector $\mathbf{b}$. 

![Diagram showing projection of vector $\mathbf{a}$ onto vector $\mathbf{b}$](image)
Projections

We apply a force to an object in the direction of $\mathbf{a}$, but we’re only concerned with the object’s movement in the direction of vector $\mathbf{b}$.

$$\mathbf{proj}_b \mathbf{a} = \left( \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{b}\|^2} \right) \mathbf{b}$$
Projections

We apply a force to an object in the direction of \( \mathbf{a} \), but we’re only concerned with the object’s movement in the direction of vector \( \mathbf{b} \).

- The vector proj\(_b\)\(\mathbf{a}\) is in the same or opposite direction as \( \mathbf{b} \).
Projections

We apply a force to an object in the direction of \( \mathbf{a} \), but we’re only concerned with the object’s movement in the direction of vector \( \mathbf{b} \).

- The vector \( \text{proj}_b \mathbf{a} \) is in the same or opposite direction as \( \mathbf{b} \).
- The vector \( \text{proj}_b \mathbf{a} \) has length \( \| \mathbf{a} \| \cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{\| \mathbf{b} \|} \).
We apply a force to an object in the direction of $\mathbf{a}$, but we’re only concerned with the object’s movement in the direction of vector $\mathbf{b}$.

- The vector $\text{proj}_b \mathbf{a}$ is in the same or opposite direction as $\mathbf{b}$.
- The vector $\text{proj}_b \mathbf{a}$ has length $\|\mathbf{a}\| \cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{b}\|}$.

\[ \text{proj}_b \mathbf{a} = \left( \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{b}\|^2} \right) \mathbf{b} \]
A man pulls a truck up a hill for some reason. If we take level ground as our coordinate axis, the hill is in the direction of the vector \[
\begin{bmatrix}
10 \\
2
\end{bmatrix},
\]
and the man applies force represented by the vector \[
\begin{bmatrix}
5 \\
2
\end{bmatrix}.
\]
What vector represents the force acting on the truck in the direction it is moving?
\[ \text{proj}_b a = \left( \frac{a \cdot b}{\|b\|^2} \right) b \]

\[ \text{proj} \begin{bmatrix} 10 \\ 2 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \left( \frac{\begin{bmatrix} 5 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 2 \end{bmatrix}}{\left\| \begin{bmatrix} 10 \\ 2 \end{bmatrix} \right\|^2} \right) \begin{bmatrix} 10 \\ 2 \end{bmatrix} \]

\[ = \left( \frac{54}{104} \right) \begin{bmatrix} 10 \\ 2 \end{bmatrix} \]

\[ = \begin{bmatrix} 540 \\ 104 \\ 108 \\ 104 \end{bmatrix} = \begin{bmatrix} 135 \\ 26 \\ 27 \\ 26 \end{bmatrix} \]
A man pulls a truck up a hill for some reason. He pulls with a force of 1000 pounds, and pulls at an angle of 20 degrees to the hill. What force is exerted in the direction of the hill? That is, what is the magnitude of the component of the force that is in the direction of the truck’s motion?

A man pulls a truck up a hill for some reason. He pulls with a force of 1000 pounds, and pulls at an angle of 20 degrees to the hill. What force is exerted in the direction of the hill? That is, what is the magnitude of the component of the force that is in the direction of the truck’s motion? $1000 \cos 20^\circ$ pounds
What is the projection of the vector \[
\begin{pmatrix}
0 \\
2 \\
5
\end{pmatrix}
\] onto the vector \[
\begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix}
\]?
What is the projection of the vector \[
\begin{bmatrix}
0 \\
2 \\
5
\end{bmatrix}
\] onto the vector \[
\begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}
\]?

A. I solved this by computing \[
\left( \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{b}\|^2} \right) \mathbf{b}
\]

B. I solved this by drawing a picture

C. I solved this by noticing that the \textit{y} component is precisely the component of the vector in the direction of \(\mathbf{j}\)

D. I solved this another way

What is the projection of the vector \[
\begin{bmatrix}
8 \\
2 \\
5
\end{bmatrix}
\] onto the vector \[
\begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}
\]?

A: still \[
\begin{bmatrix}
0 \\
2 \\
0
\end{bmatrix}
\]

B: probably not

What is the projection of the vector \[
\begin{bmatrix}
8 \\
2 \\
5
\end{bmatrix}
\] onto the vector \[
\begin{bmatrix}
1 \\
2 \\
3
\end{bmatrix}
\]?

\[
\begin{bmatrix}
\frac{27}{14} \\
\frac{27}{7} \\
81/14
\end{bmatrix}
\]

What is the projection of the vector \(\mathbf{a}\) onto itself? \(\mathbf{a}\)
What is the projection of the vector \[ \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix} \] onto the vector \[ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \]?

What is the projection of the vector \[ \begin{bmatrix} 8 \\ 2 \\ 5 \end{bmatrix} \] onto the vector \[ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \]?

A: still \[ \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \]  
B: probably not \[ \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \] any more
What is the projection of the vector \[
\begin{bmatrix}
0 \\
2 \\
5
\end{bmatrix}
\] onto the vector \[
\begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}
\]?

What is the projection of the vector \[
\begin{bmatrix}
8 \\
2 \\
5
\end{bmatrix}
\] onto the vector \[
\begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}
\]?

A: still \[
\begin{bmatrix}
0 \\
2 \\
0
\end{bmatrix}
\]

B: probably not \[
\begin{bmatrix}
0 \\
2 \\
0
\end{bmatrix}
\] any more
What is the projection of the vector \([0 \\ 2 \\ 5]\) onto the vector \([0 \\ 1 \\ 0]\)?

What is the projection of the vector \([8 \\ 2 \\ 5]\) onto the vector \([1 \\ 2 \\ 3]\)?
What is the projection of the vector \[
\begin{bmatrix}
0 \\
2 \\
5
\end{bmatrix}
\] onto the vector \[
\begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}
\]?

What is the projection of the vector \[
\begin{bmatrix}
8 \\
2 \\
5
\end{bmatrix}
\] onto the vector \[
\begin{bmatrix}
1 \\
2 \\
3
\end{bmatrix}
\]?

\[
\begin{bmatrix}
27/14 \\
27/7 \\
81/14
\end{bmatrix}
\]
What is the projection of the vector \[
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0 \\
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5
\end{bmatrix}
\] onto the vector \[
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0 \\
1 \\
0
\end{bmatrix}
\]?

What is the projection of the vector \[
\begin{bmatrix}
8 \\
2 \\
5
\end{bmatrix}
\] onto the vector \[
\begin{bmatrix}
1 \\
2 \\
3
\end{bmatrix}
\]?

\[
\begin{bmatrix}
27/14 \\
27/7 \\
81/14
\end{bmatrix}
\]

What is the projection of the vector \( \mathbf{a} \) onto itself?
What is the projection of the vector \[
\begin{bmatrix}
0 \\
2 \\
5
\end{bmatrix}
\] onto the vector \[
\begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}
\]?

What is the projection of the vector \[
\begin{bmatrix}
8 \\
2 \\
5
\end{bmatrix}
\] onto the vector \[
\begin{bmatrix}
1 \\
2 \\
3
\end{bmatrix}
\]?

\[
\begin{bmatrix}
27/14 \\
27/7 \\
81/14
\end{bmatrix}
\]

What is the projection of the vector \( \mathbf{a} \) onto itself?  \( \mathbf{a} \)