1. Find eigenvalues

\[
\text{det}(A - \lambda I) = \text{det}\begin{pmatrix}
-\lambda & 1 & \lambda \\
-1 & 1-\lambda & 0 \\
-\lambda & 0 & -\lambda
\end{pmatrix}
\]

\[
-\lambda \text{det}\begin{pmatrix}
1-\lambda & 1 \\
-1 & 1-\lambda
\end{pmatrix} = -\lambda \left[ (1-\lambda)^2 + 1 \right] = 0
\]

\[\lambda = 0\] or \[(1-\lambda)^2 + 1 = 0\]

\[(1-\lambda)^2 = -1\]

\[1-\lambda = \pm \sqrt{-1} = \pm i\]

\[1 \pm i = \lambda\]
2. Find eigenvectors

\[ \lambda = 0 \]

\[
\begin{pmatrix}
1 & 1 & 0 \\
-1 & 1 & c \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}
\]

\[
x + y = 0 \\
-x + y = 0 \\
0 = 0
\]

\[ 2y = 0, \text{ so } y = 0 \]

\[ x + y = 0 \rightarrow x = 0 \]

z: anything

\[ x_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \]

\[ \lambda = 1+i \]

\[
\begin{pmatrix}
1 & 1 & 0 \\
-1 & 1 & c \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
= 
(1+i)
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
\]

\[ x + y = x + ix ightarrow y = ix \quad \text{equiv.} \]

\[ -x + y = y + iy ightarrow -x = iy \quad \text{(mult by } i \text{)} \]

\[ 0 = (1+i)z \rightarrow z = 0 \]

\[ x_2 = \begin{bmatrix} 1 \\ i \\ 0 \end{bmatrix} \]

Ignore \[ \lambda = 1-i; \quad x_3 = \begin{bmatrix} 1 \\ -i \\ 0 \end{bmatrix} \]
General Solution:

\[ c_1 e^{\lambda_1 t} x_1 + c_2 e^{\lambda_2 t} x_2 + c_3 e^{\lambda_3 t} x_3 \]

\[ \lambda_1 = 0 \]

Instead of \( J \), use:

\[ c_2 \text{Re}(e^{\lambda_2 t} x_2) + c_3 \text{Im}(e^{\lambda_2 t} x_2) \]

\[ e^{\lambda_2 t} x_2 = e^{(1+i)t} \begin{bmatrix} 1 \\ i \end{bmatrix} = e^t e^{it} \begin{bmatrix} 1 \\ i \end{bmatrix} \]

\[ = e^t (\cos t + is \sin t) \begin{bmatrix} 1 \\ i \end{bmatrix} \]

\[ = \begin{bmatrix} e^t \cos t + ie^t \sin t \\ ie^t \cos t - e^t \sin t \end{bmatrix} = \begin{bmatrix} e^t \cos t \\ -e^t \sin t \end{bmatrix} + i \begin{bmatrix} e^t \sin t \\ e^t \cos t \end{bmatrix} \]

\[ \text{Re} \]

\[ \text{Im} \]
General Solution:
\[ C_1 e^{\lambda_1 t} [\mathbf{0}] + C_2 \begin{bmatrix} e^{t \cos t} \\ -e^{t \sin t} \end{bmatrix} + C_3 \begin{bmatrix} e^{t \sin t} \\ e^{t \cos t} \end{bmatrix} \]

\[ \lambda_1 = 0 \]
\[ x_1 = [\mathbf{0}] \]
\[ x_1 = [\mathbf{0}] \]