Math 105, Section 204, Quiz 2
Wednesday, January 24

Communicating mathematics is an important skill to practice, so simplify and justify all answers unless otherwise directed. Show your work, and use proper notation.

Second Derivative Test:
Let $D(x, y) = f_{xx}(x, y)f_{yy}(x, y) - f_{xy}^2(x, y)$, where $f(x, y)$ is a function whose second partial derivatives are continuous throughout an open disk centred at the point (a, b), and $f_x(a, b) = f_y(a, b) = 0$.

1. If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$, then f has a local maximum value at (a, b).
2. If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then f has a local minimum value at (a, b).
3. If $D(a, b) < 0$, then f has a saddle point at (a, b).

1. A surface is described by the equation $\mathbf{z = (x+2)^2y^3 + y^2 + 2y + 4}$ where z is the vertical axis (as usual). You drop a ball onto the surface, and it settles into the point (a, b). Give all possible values of (a, b). $\boxed{5}$

2. Someone is trying to sketch the level curves of the surface defined by the equation $\mathbf{z = e^{x^2} - y^2}$ over the closed region $\mathbf{R = \{(x, y) \ : \ x^2 + y^2 \leq 4\}}$. They notice that some constant values of z give no solutions over R. $\boxed{5}$

   Find the range constant values of z that will have solutions in R. That is, find the largest and smallest values of z so that there exists a point (x, y) in R with $z = e^{x^2} - y^2$.

   Hint: by the chain rule, $\frac{\mathrm{d}}{\mathrm{d}x}\left[e^{x^2}\right] = 2xe^{x^2}$.

3. Double or Nothing **(optional)**

   Pick one of the two questions on this quiz. Full marks will be doubled; partial marks will be deleted.

   $\boxed{}$

   The purpose of this question is to encourage you to develop self-reliance in evaluating your understanding.